

A Scaled Poisson Bayesian Model for Viral Epidemic Monitoring

B. Pascal^{1,†}, P. Abry^{2,†}, J. Chevallier^{3,†}, G. Fort^{4,†}

GdR IASIS

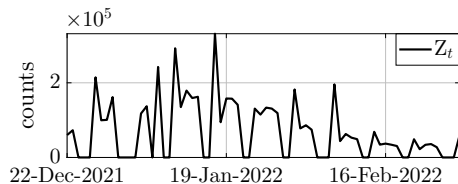
Bayesian inference for inverse problems

June 11, 2026

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1. CNRS, LS2N, Nantes, France
 2. CNRS, ENS de Lyon, Laboratoire de Physique, France,
 3. IMT, INSA Toulouse, France,
 4. LAAS-CNRS, CNRS, Toulouse, France

Main challenges of epidemic surveillance

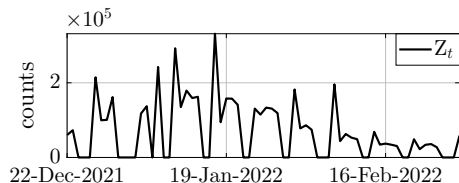
Daily counts of new COVID-19 infections in Spain during 10 weeks



data collected
by **Johns Hopkins University**
from Public Health Agencies

Main challenges of epidemic surveillance

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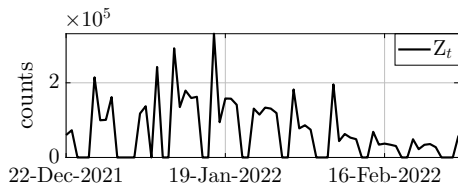
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Design and evaluation of timely and adapted sanitary control measures requires:

- efficient monitoring tools based on sound **epidemiological model**,
- robustness to low quality of the data, i.e., **capacity to handle reporting errors**,
- reliable confidence levels through **credibility intervals**.

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Key indicator: reproduction number R_0

(Ferguson et al., 2006, *Nature*)

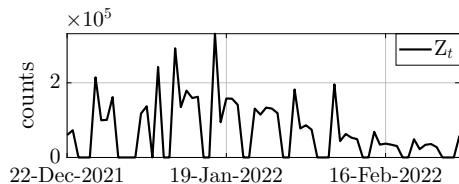
“averaged number of secondary cases generated by a typical contagious individual”

⇒ relaxed into an **effective time-varying reproduction number** R_t at day t

$R_t > 1$: **exponential** increase of infections

(Cori et al., 2013, *Am. J. Epidemiol.*)

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Infection counts model:

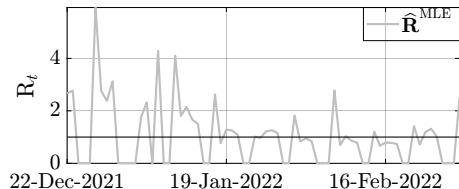
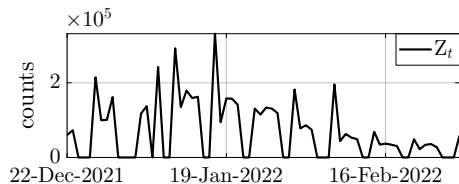
$$\mathbb{P}(Z_t | Z_{t-1}, Z_{t-2}, \dots) = \text{Poisson}(p_t),$$

$$\text{with } p_t = R_t \sum_{u=1}^{\tau} \phi_u Z_{t-u} = R_t \Phi_t$$

$\{\phi_u\}_{u=1}^{\tau}$: serial interval distribution

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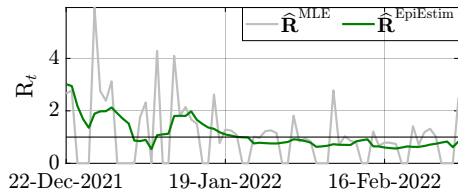
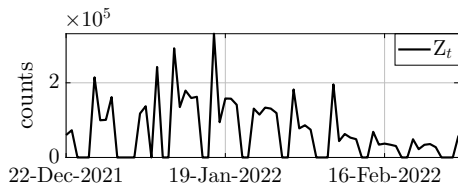
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Reference R_t estimators:

Maximum Likelihood Estimate: (MLE)

$$R_t = Z_t / \Phi_t$$

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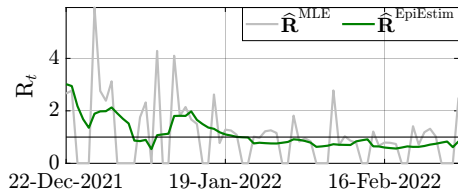
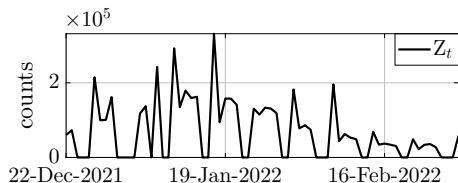
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EpiEstim: $a = 1$ and $1/b = 1/5$

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temporal smoothing over $\tau' = 7$ days

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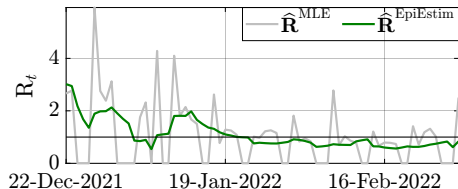
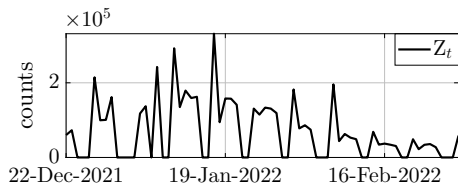
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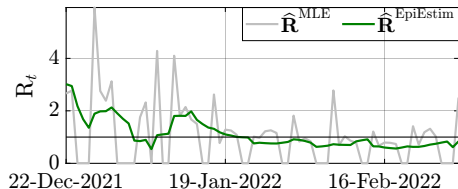
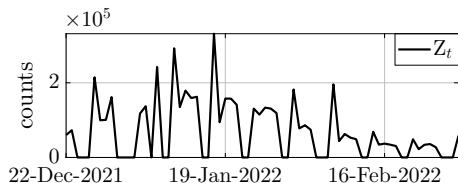
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Epidemic monitoring during T days:

- observations: $\mathbf{Z} = (Z_1, \dots, Z_T)$ infection counts
- unknown parameters: $\mathbf{R} = (R_1, \dots, R_T)$ time-varying reproduction number

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Prior distributions of unknown parameters and hidden variables:

$$(\mathbf{DR})_t := (R_t - 2R_{t-1} + R_{t-2})/4 \sim \text{Laplace}(\lambda_R), \quad O_t \sim \text{Laplace}(\lambda_O);$$

$\underline{\lambda} := (\lambda_R, \lambda_O)$: positive hyperparameters

- R_1, \dots, R_T few abrupt slope changes: **temporal regularity**
- O_1, \dots, O_T independent random variables: **error sparsity**

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$(\mathbf{Z}, \mathbf{R}, \mathbf{O})$ **log-density**: $\ell(\mathbf{Z} | \mathbf{R}, \mathbf{O}, \mathcal{I}) - \lambda_R \|\mathbf{DR} + \delta\| - \lambda_O \|\mathbf{O}\|$ $\mathcal{I} := (\mathbf{Z}_{-\tau+1:0}, R_{-1}, R_0)$

provided that $R_t \geq 0$ and $R_t \Phi_t + O_t \geq 0$ for all t , otherwise $-\infty$;

$$\ell(\mathbf{Z} | \mathbf{R}, \mathbf{O}, \mathcal{I}) := \sum_{t=1}^T \{Z_t \ln(R_t \Phi_t + O_t) - (R_t \Phi_t + O_t)\}; \delta := ((-2R_0 + R_{-1})/4, R_0/4, 0, \dots, 0)$$

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Hyperparameter distribution: allow $\underline{\lambda} := (\lambda_R, \lambda_O)$ to explore many values

product of independent **conjugate gamma** priors $\mathcal{G}(\alpha_R, \beta_R) \otimes \mathcal{G}(\alpha_O, \beta_O)$

α_R, α_O and β_R, β_O shape and rate parameters respectively (Abry et al., 2025, ICASSP)

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Log-density of $(\mathbf{Z}, \mathbf{R}, \mathbf{O}, \underline{\lambda})$: $\ell(\mathbf{Z}|\mathbf{R}, \mathbf{O}, \mathcal{I}) + \text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda})$ with $\text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda}) :=$

$$-\lambda_R (\beta_R + \|\mathbf{D}\mathbf{R} + \delta\|_1) - \lambda_O (\beta_O + \|\mathbf{O}\|_1) + (T + \alpha_R - 1) \ln \lambda_R + (T + \alpha_O - 1) \ln \lambda_O$$

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Marginal distribution: $\bar{\pi}(\mathbf{R}, \mathbf{O}|\mathbf{Z}, \mathcal{I}) = \int_{\mathbb{R}_{>0} \times \mathbb{R}_{>0}} \pi(\mathbf{R}, \mathbf{O}, \underline{\lambda}|\mathbf{Z}, \mathcal{I}) d\underline{\lambda}$

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with $\exp(\overline{\text{pen}}(\mathbf{R}, \mathbf{O})) \propto (\beta_R + \|\mathbf{D}\mathbf{R} + \delta\|_1)^{-(T+\alpha_R)} (\beta_O + \|\mathbf{O}\|_1)^{-(T+\alpha_O)}$

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Gibbs sampler: Markov chain Monte Carlo algorithm targeting $\bar{\pi}$

Adjusted Proximal-Langevin mechanism PGdual to sample $\mathbf{R}, \mathbf{O}|\underline{\lambda}$

Exact conditional sampling of $\underline{\lambda}|\mathbf{R}, \mathbf{O}$ thanks to conjugate priors

(Fort et al., 2023, IEEE Trans. Signal Process.; Abry et al., 2025, ICASSP)

Hierarchical estimates: $k_{\max} = 5 \cdot 10^7$ including $k_{\text{burn-in}} = 3 \cdot 10^7$ iterations

$$\hat{\mathbf{R}}^{\text{Hierarchical}} := \frac{1}{K} \sum_{k=k_{\text{burn-in}}}^{k_{\max}} \mathbf{R}^{(k)}, \quad \mathbf{z} - \hat{\mathbf{O}}^{\text{Hierarchical}} := \mathbf{z} - \frac{1}{K} \sum_{k=k_{\text{burn-in}}}^{k_{\max}} \mathbf{O}^{(k)}$$

and **95% credibility intervals** from 2.5% and 97.5% empirical quantiles

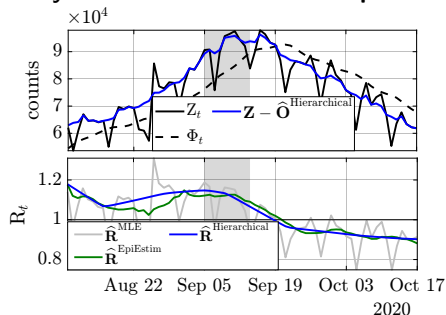
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Daily surveillance of COVID-19 pandemic in India during 10 weeks



accurate mean a posteriori estimates

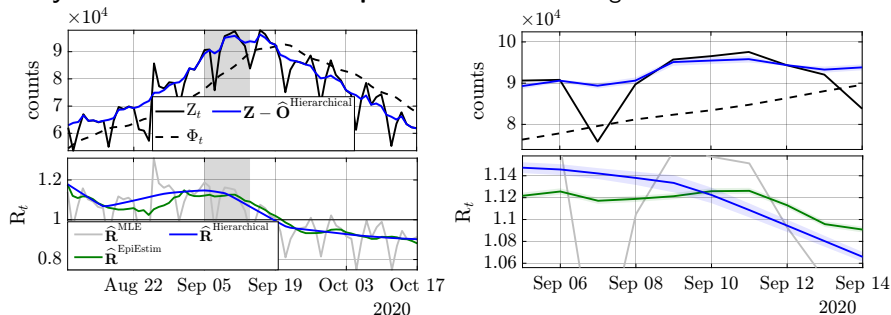
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accurate mean a posteriori estimates but **unrealistically narrow** credibility interval

Tempered likelihood:

$$\bar{\pi}_\nu(\mathbf{R}, \mathbf{O}|\mathbf{Z}, \mathcal{I}) \propto \exp\left(\frac{1}{\nu}\ell(\mathbf{Z}|\mathbf{R}, \mathbf{O}, \mathcal{I}) + \overline{\text{pen}}(\mathbf{R}, \mathbf{O})\right)$$

\implies unbalances likelihood vs. priors

- $\nu = 1$ standard Poisson
- $\nu = \infty$ prior distribution
- $\nu > 1$ count **over-dispersion**

In practice: $\nu \propto \text{std}(\mathbf{Z})$

Scaled-likelihood accounting for model misspecification

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Markov Chain Monte Carlo algorithm:

Input: $\mathbf{Z}, \Phi, \mathcal{I}, \nu$;

Parameters: $(\alpha_R, \beta_R, \alpha_O, \beta_O), k_{\max}$;

Initialize: $(\mathbf{R}^{(0)}, \mathbf{O}^{(0)}, \underline{\lambda}^{(0)})$;

for $k = 0, 1, \dots, k_{\max} - 1$ **do**

 # Sample \mathbf{R} then \mathbf{O} at fixed $\lambda_R^{(k)}, \lambda_O^{(k)}$

$\mathbf{R}^{(k+1)}, \mathbf{O}^{(k+1)} \sim \nu\text{-PGdual}(\mathbf{R}^{(k)}, \mathbf{O}^{(k)}; \lambda_R^{(k)}, \lambda_O^{(k)})$;

 # Sample λ_R then λ_O at fixed $\mathbf{R}^{(k+1)}, \mathbf{O}^{(k+1)}$

$\lambda_R^{(k+1)} \sim \mathcal{G}(T + \alpha_R, \|\mathbf{D}\mathbf{R}^{(k+1)} + \delta\| + \beta_R)$;

$\lambda_O^{(k+1)} \sim \mathcal{G}(T + \alpha_O, \|\mathbf{O}^{(k+1)}\| + \beta_O)$;

end for

Output: $\{\mathbf{R}^{(k)}, \mathbf{O}^{(k)}, \lambda_R^{(k)}, \lambda_O^{(k)}\}_{k=1, \dots, k_{\max}}$

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Output: $\{\mathbf{R}^{(k)}, \mathbf{O}^{(k)}, \lambda_R^{(k)}, \lambda_O^{(k)}\}_{k=1, \dots, k_{\max}}$

Proposed ν -scaled estimates: $k_{\max} = 5 \cdot 10^7$ including $k_{\text{burn-in}} = 3 \cdot 10^7$ iterations

$$\widehat{\mathbf{R}}^{\nu\text{-scaled}} := \frac{1}{K} \sum_{k=k_{\text{burn-in}}}^{k_{\max}} \mathbf{R}^{(k)}, \quad \mathbf{Z} - \widehat{\mathbf{O}}^{\nu\text{-scaled}} := \mathbf{Z} - \frac{1}{K} \sum_{k=k_{\text{burn-in}}}^{k_{\max}} \mathbf{O}^{(k)}$$

and 95% **credibility intervals** from 2.5% and 97.5% empirical quantiles

Log-density of $(\mathbf{Z}, \mathbf{R}, \mathbf{O}, \underline{\lambda})$: $\nu^{-1} \ell(\mathbf{Z} | \mathbf{R}, \mathbf{O}, \mathcal{I}) + \text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda})$ with $\text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda}) :=$

$$-\lambda_{\mathbf{R}} (\beta_{\mathbf{R}} + \|\mathbf{D}\mathbf{R} + \delta\|_1) - \lambda_{\mathbf{O}} (\beta_{\mathbf{O}} + \|\mathbf{O}\|_1) + (T + \alpha_{\mathbf{R}} - 1) \ln \lambda_{\mathbf{R}} + (T + \alpha_{\mathbf{O}} - 1) \ln \lambda_{\mathbf{O}}$$

including conjugate gamma priors on $\underline{\lambda} = (\lambda_{\mathbf{R}}, \lambda_{\mathbf{O}}) \sim \mathcal{G}(\alpha_{\mathbf{R}}, \beta_{\mathbf{R}}) \otimes \mathcal{G}(\alpha_{\mathbf{O}}, \beta_{\mathbf{O}})$

Log-density of $(\mathbf{Z}, \mathbf{R}, \mathbf{O}, \underline{\lambda})$: $\nu^{-1} \ell(\mathbf{Z}|\mathbf{R}, \mathbf{O}, \mathcal{I}) + \text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda})$ with $\text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda}) :=$

$$-\lambda_{\mathbf{R}} (\beta_{\mathbf{R}} + \|\mathbf{DR} + \delta\|_1) - \lambda_{\mathbf{O}} (\beta_{\mathbf{O}} + \|\mathbf{O}\|_1) + (T + \alpha_{\mathbf{R}} - 1) \ln \lambda_{\mathbf{R}} + (T + \alpha_{\mathbf{O}} - 1) \ln \lambda_{\mathbf{O}}$$

including conjugate gamma priors on $\underline{\lambda} = (\lambda_{\mathbf{R}}, \lambda_{\mathbf{O}}) \sim \mathcal{G}(\alpha_{\mathbf{R}}, \beta_{\mathbf{R}}) \otimes \mathcal{G}(\alpha_{\mathbf{O}}, \beta_{\mathbf{O}})$

Scaled marginal distribution: $\ln \bar{\pi}_{\nu}(\mathbf{R}, \mathbf{O}|\mathbf{Z}, \mathcal{I}) = \nu^{-1} \ell(\mathbf{Z}|\mathbf{R}, \mathbf{O}, \mathcal{I}) + \overline{\text{pen}}(\mathbf{R}, \mathbf{O})$ with

$$\exp(\overline{\text{pen}}(\mathbf{R}, \mathbf{O})) \propto (\beta_{\mathbf{R}} + \|\mathbf{DR} + \delta\|_1)^{-(T+\alpha_{\mathbf{R}})} (\beta_{\mathbf{O}} + \|\mathbf{O}\|_1)^{-(T+\alpha_{\mathbf{O}})}$$

Log-density of $(\mathbf{Z}, \mathbf{R}, \mathbf{O}, \underline{\lambda})$: $\nu^{-1} \ell(\mathbf{Z}|\mathbf{R}, \mathbf{O}, \mathcal{I}) + \text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda})$ with $\text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda}) :=$
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Quasi-noninformative priors on $\underline{\lambda} = (\lambda_{\mathbf{R}}, \lambda_{\mathbf{O}})$: gamma distributions

- means $\alpha_{\mathbf{R}}/\beta_{\mathbf{R}} = 3.5 \times \text{std}(\mathbf{Z})$
 $\alpha_{\mathbf{O}}/\beta_{\mathbf{O}} = 5 \times 10^{-2}$ (Pascal et al., 2022, IEEE Trans. Signal Process)

Log-density of $(\mathbf{Z}, \mathbf{R}, \mathbf{O}, \underline{\lambda})$: $\nu^{-1} \ell(\mathbf{Z}|\mathbf{R}, \mathbf{O}, \mathcal{I}) + \text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda})$ with $\text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda}) :=$
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including conjugate gamma priors on $\underline{\lambda} = (\lambda_{\mathbf{R}}, \lambda_{\mathbf{O}}) \sim \mathcal{G}(\alpha_{\mathbf{R}}, \beta_{\mathbf{R}}) \otimes \mathcal{G}(\alpha_{\mathbf{O}}, \beta_{\mathbf{O}})$

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Quasi-noninformative priors on $\underline{\lambda} = (\lambda_{\mathbf{R}}, \lambda_{\mathbf{O}})$: gamma distributions

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 $\alpha_{\mathbf{O}}/\beta_{\mathbf{O}} = 5 \times 10^{-2}$
- standard deviations $\sqrt{\alpha_{\mathbf{R}}}/\beta_{\mathbf{R}} = 50 \times (\alpha_{\mathbf{R}}/\beta_{\mathbf{R}})$, $\sqrt{\alpha_{\mathbf{O}}}/\beta_{\mathbf{O}} = 80 \times (\alpha_{\mathbf{O}}/\beta_{\mathbf{O}})$

Log-density of $(\mathbf{Z}, \mathbf{R}, \mathbf{O}, \underline{\lambda})$: $\nu^{-1} \ell(\mathbf{Z}|\mathbf{R}, \mathbf{O}, \mathcal{I}) + \text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda})$ with $\text{pen}(\mathbf{R}, \mathbf{O}, \underline{\lambda}) :=$
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including conjugate gamma priors on $\underline{\lambda} = (\lambda_{\mathbf{R}}, \lambda_{\mathbf{O}}) \sim \mathcal{G}(\alpha_{\mathbf{R}}, \beta_{\mathbf{R}}) \otimes \mathcal{G}(\alpha_{\mathbf{O}}, \beta_{\mathbf{O}})$

Scaled marginal distribution: $\ln \bar{\pi}_{\nu}(\mathbf{R}, \mathbf{O}|\mathbf{Z}, \mathcal{I}) = \nu^{-1} \ell(\mathbf{Z}|\mathbf{R}, \mathbf{O}, \mathcal{I}) + \overline{\text{pen}}(\mathbf{R}, \mathbf{O})$ with
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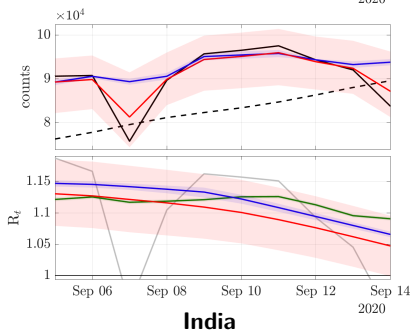
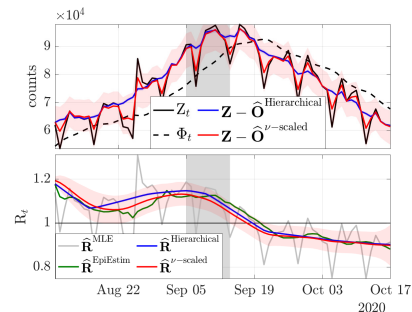
Quasi-noninformative priors on $\underline{\lambda} = (\lambda_{\mathbf{R}}, \lambda_{\mathbf{O}})$: gamma distributions

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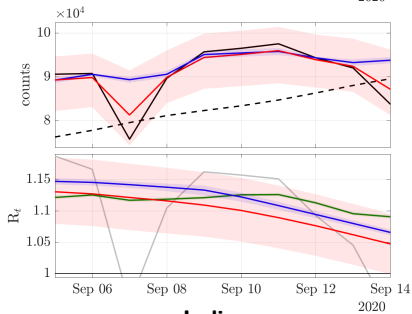
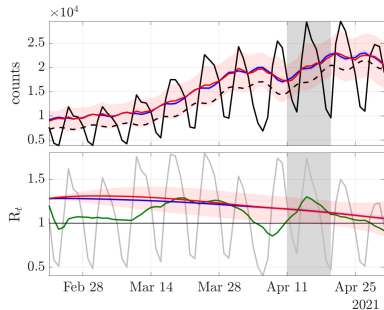
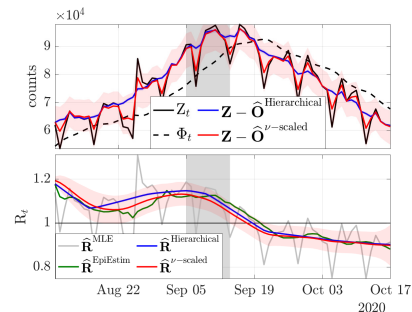
Hierarchical estimator, i.e., $\nu = 1$: informative gamma priors on $\underline{\lambda}$

- means $\alpha_{\mathbf{R}}/\beta_{\mathbf{R}} = 3.5 \times \text{std}(\mathbf{Z})$ (Pascal et al., 2022, IEEE Trans. Signal Process)
 $\alpha_{\mathbf{O}}/\beta_{\mathbf{O}} = 5 \times 10^{-2}$
- standard deviations $\sqrt{\alpha_{\mathbf{R}}}/\beta_{\mathbf{R}} = 0.02 \times (\alpha_{\mathbf{R}}/\beta_{\mathbf{R}})$, $\sqrt{\alpha_{\mathbf{O}}}/\beta_{\mathbf{O}} = 0.0125 \times (\alpha_{\mathbf{O}}/\beta_{\mathbf{O}})$
(Abry et al., 2025, ICASSP)

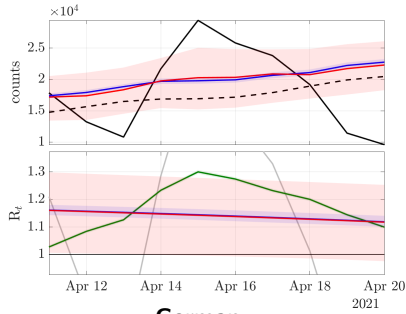
Robust estimation of reproduction number credibility intervals



Robust estimation of reproduction number credibility intervals

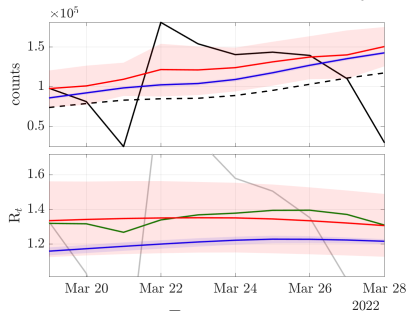
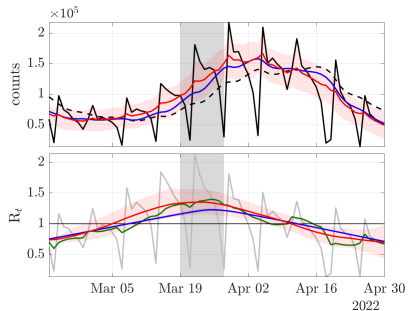


India



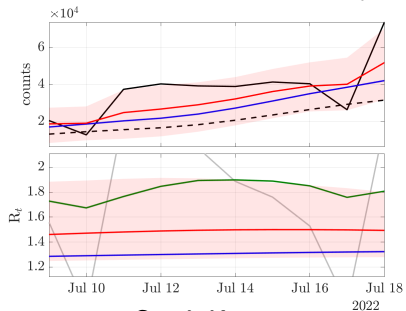
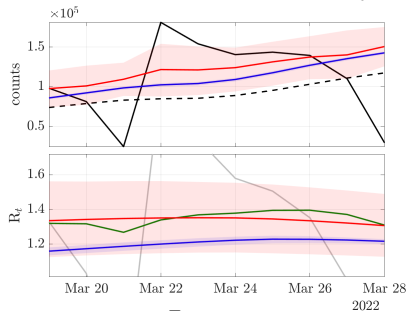
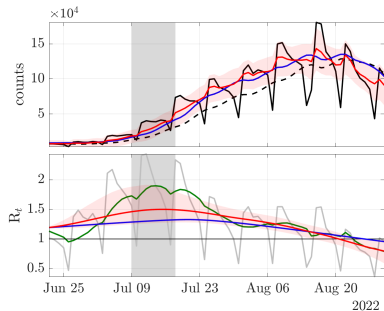
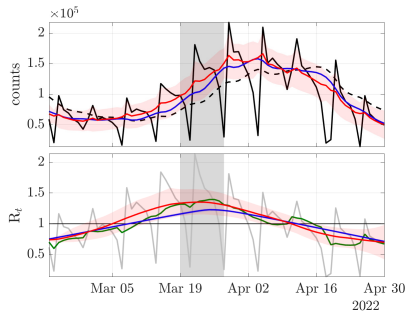
Germany

Robust estimation of reproduction number credibility intervals



France

Robust estimation of reproduction number credibility intervals

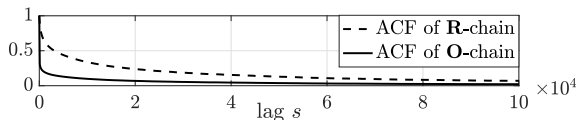


France

South Korea

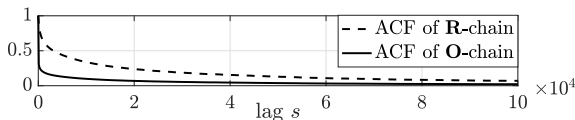
Absolute value of autocorrelation averaged across the T components:

- $R_t^{(0)} = (R_{t,7}^{\text{EpiEstim}} + 1)/2$, $O_t^{(0)} = (Z_t - R_{t,7}^{\text{EpiEstim}} \Phi_t)/2$
- $\lambda_R^{(0)} = \alpha_R/\beta_R$
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Area covered by reproduction number credibility intervals

- averaged over **10 runs** of the Gibbs sampler
- accompanied with 95% Gaussian confidence intervals

	India	Germany	France	South Korea
EpiEstim (Cori et al., 2013, Am. J. Epidemiol.)	0.4	1.1	0.4	0.8
Hierarchical (Abry et al., 2025, ICASSP)	$0.6 \pm 0.0^*$	$2.1 \pm 0.0^*$	1.4 ± 0.1	$0.9 \pm 0.0^*$

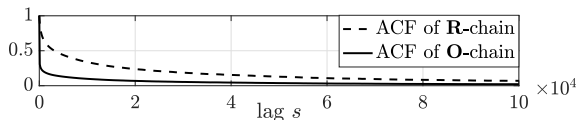
ν -scaled

$\nu = 0.025 \times \text{std}(\mathbf{Z})$	7.6 ± 0.1	18.3 ± 0.2	22.6 ± 0.4	22.6 ± 1.0
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* 95% confidence intervals smaller than 0.1

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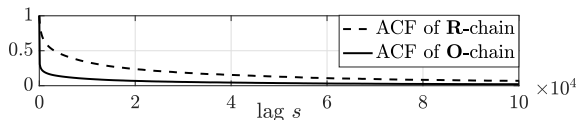
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ν-scaled				
$\nu = 0.0125 \times \text{std}(\mathbf{Z})$	7.2 ± 0.1	18.2 ± 0.7	21.5 ± 0.3	22.1 ± 1.2
$\nu = 0.025 \times \text{std}(\mathbf{Z})$	7.6 ± 0.1	18.3 ± 0.2	22.6 ± 0.4	22.6 ± 1.0

* 95% confidence intervals smaller than 0.1

Convergence and robustness to the selection of the scale parameter

Absolute value of autocorrelation averaged across the T components:

- $R_t^{(0)} = (R_{t,7}^{\text{EpiEstim}} + 1)/2$, $O_t^{(0)} = (Z_t - R_{t,7}^{\text{EpiEstim}} \Phi_t)/2$
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Area covered by reproduction number credibility intervals

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	India	Germany	France	South Korea
EpiEstim (Cori et al., 2013, Am. J. Epidemiol.)	0.4	1.1	0.4	0.8
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ν-scaled				
$\nu = 0.0125 \times \text{std}(\mathbf{Z})$	7.2 ± 0.1	18.2 ± 0.7	21.5 ± 0.3	22.1 ± 1.2
$\nu = 0.025 \times \text{std}(\mathbf{Z})$	7.6 ± 0.1	18.3 ± 0.2	22.6 ± 0.4	22.6 ± 1.0
$\nu = 0.05 \times \text{std}(\mathbf{Z})$	8.2 ± 0.1	19.0 ± 0.6	23.4 ± 0.6	22.6 ± 0.8

* 95% confidence intervals smaller than 0.1

Contributions and take home messages

- Bayesian modeling of epidemic propagation accounting for **low quality** counts
- Introduction of a **scale** parameter ν to account for count **over-dispersion**
- Efficient original **Gibbs** sampler adapted to the **hierarchical** model
- Uncertainty quantification through estimation of **credibility intervals**
- **Robustness** of the estimates to model misspecification

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Accurate estimates and **trustworthy** credibility intervals \implies **usable by nonspecialists**

- ▶ improvement in accuracy compared to reference methods
- ▶ more realistic credibility intervals than previously designed Bayesian estimates
- ▶ quantified convergence of the MCMC algorithm and robustness to choice of ν

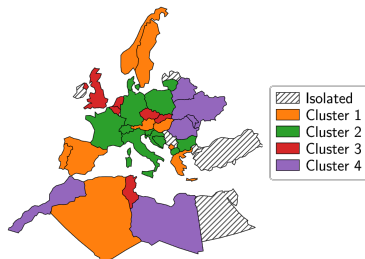


Matlab toolbox publicly available at github.com/gfort-lab/OpSiMorE

- ▶ Decoupling of the stochastic **pathogen propagation** and **administrative noise**

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- ▶ **Spatiotemporal** pandemic monitoring

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- ▶ **Spatiotemporal** pandemic monitoring: for the moment
 - pointwise estimates, not yet uncertainty quantifications
 - data-driven inference of the spatial structure



OptiMoCSI project funded by ANR optimocsi.cnrs.fr