Texture segmentation based on multiscale parameters

Combining local variance and local regularity into an optimization scheme

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Aims and contributions

Texture segmentation constitutes a task of utmost importance in statistical image processing. Monofractal textures, characterized by piecewise constancy of their scale-free parameters, were recently shown to be versatile enough for real-world texture modeling.

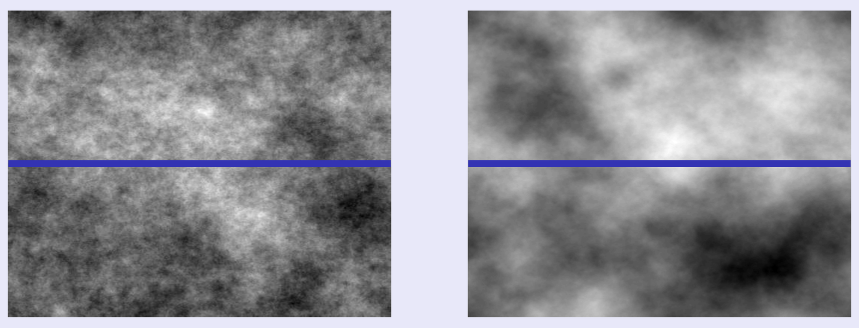
Contributions
- enrolling jointly scale-free and local-variance descriptors into a convex, but nonsmooth, minimization strategy,
- designing an efficient implementation, able to deal with huge amounts of data/high resolution images.

Numerical assessment Performance of the proposed joint approach is compared against disjoint strategies working independently on scale-free features and on local variance on synthetic piecewise monofractal textures.

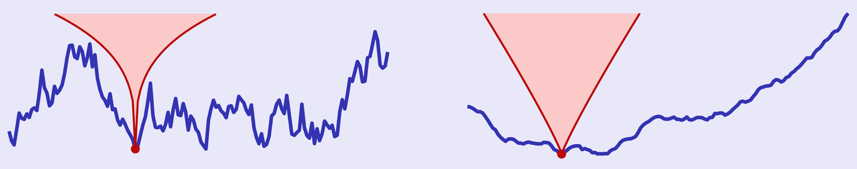
Local regularity

➔ measures how regular the texture is.

[Jaffard1994]



$h = 0.3$ $h = 0.9$



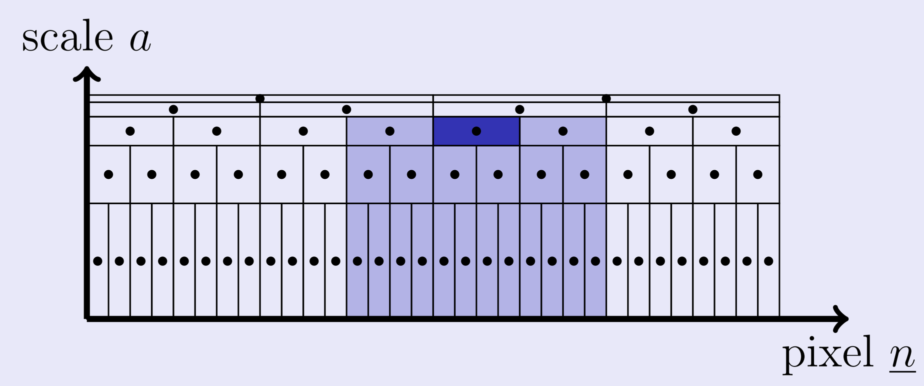
Fit local behavior with **power law** functions

$$|f(x) - f(y)| \leq C|x-y|^{h(x)}$$

Multiscale analysis

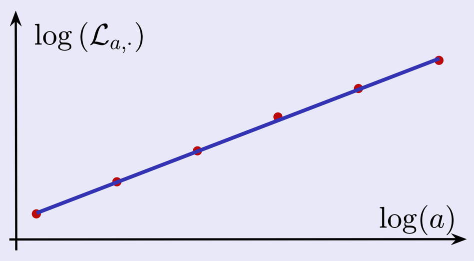
- Compute wavelet coefficients of the considered textured image X, at scale a and pixel $\underline{n}$.
- Define *leaders* $\mathcal{L}_{a,\underline{n}}$ as a local supremum within a spatial neighborhood and finer scales.
- In logarithmic coordinates *leaders* show linear behavior w.r.t. the scale.

***Leaders* coefficients**



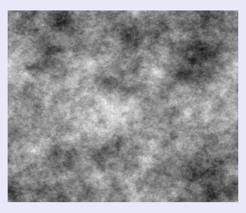
Linear regression

$$\log\left(\mathcal{L}_{a,\underline{n}}\right) \simeq v(\underline{n}) + h(\underline{n})\log(a) \qquad \textbf{(LR)}$$



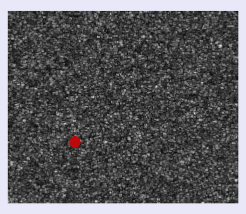
[Wendt2009]

From texture X



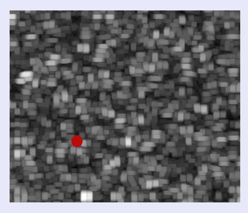
Compute *leaders* at each scale

Scales $a = 2^1$



...

Scale $a = 2^6$



Optimization scheme

Objectives:
- match scale-free behavior **(LR)** characterizing monofractal textures,
- obtain a joint estimation of v and h directly from the *leaders* $\mathcal{L}_{a,.}$,
- favor piecewise constancy of v and h without imposing same edges for v and h.

Minimization problem:

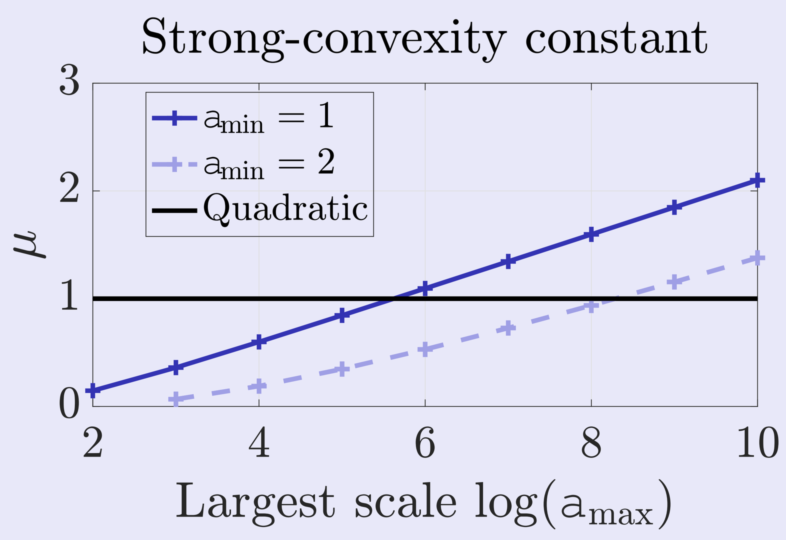
$$\left(\widehat{v}, \widehat{h}\right) \in \underset{v,h}{\operatorname{Argmin}}\ \underbrace{\mathbf{DF}(v, h\,;\,\mathcal{L}(X))}_{\text{Monofractal texture}} + \underbrace{\lambda_v \mathbf{TV}(v) + \lambda_h \mathbf{TV}(h)}_{\text{Piecewise constancy}}$$

Proposed data fidelity term:

$$\mathbf{DF}(v, h\,;\,\mathcal{L}) = \frac{1}{2}\sum_{a_{\min}}^{a_{\max}} \|v + \log(a)h - \log \mathcal{L}_{a,.}\|_2^2$$

DF is strongly convex:

$$\mathbf{DF}(v, h\,;\,\mathcal{L}) = \frac{1}{2}\|\mathbf{A}(v,h) - \log \mathcal{L}\|_2^2$$



with $\mathbf{A} : (v,h) \mapsto \{v + \log(a)h\}_a$ **linear**. Then

$$\nabla\mathbf{DF}(v, h\,;\,\mathcal{L}) = \underbrace{\mathbf{A}^*\mathbf{A}(v,h)}_{\text{linear}} - \underbrace{\mathbf{A}^* \log \mathcal{L}}_{\text{constant}}.$$

$\mathbf{DF}(v, h\,;\,\mathcal{L})$ is μ-strongly-convex,

with $\mu > 0$ the smallest eigenvalue of $\mathbf{A}^*\mathbf{A}$

Acc. primal-dual alg.

[Chambolle2011]

Customized for our objective function

Primal var. $vh \underset{\text{def}}{\equiv} (v,h)$, dual var. $u\ell \underset{\text{def}}{\equiv} (u,\ell)$

for $k \in \mathbb{N}^*$ **do**

```
// Update of primal variable
```
$$vh^{[k+1]} = \operatorname{prox}_{\delta_k \mathbf{DF}(.,\mathcal{L})}\left(vh^{[k]} - \delta_k \mathbf{D}^* \overline{u\ell}^{[k]}\right)$$
```
// Update of dual variable
```
$$u\ell^{[k+1]} = \operatorname{prox}_{\nu_k \Lambda \|\cdot\|_{2,1}^*}\left(u\ell^{[k]} + \nu_k \mathbf{D} vh^{[k]}\right)$$
```
// Update of descent steps
```
$$\vartheta_k = (1 + 2\mu\delta_k)^{-1/2},$$
$$\underbrace{\delta_{k+1} = \vartheta_k \delta_k}_{\text{smaller}}, \quad \underbrace{\nu_{k+1} = \nu_k/\vartheta_k}_{\text{larger}}$$
```
// Update of auxiliary variable
```
$$\overline{u\ell}^{[k+1]} = u\ell^{[k+1]} + \vartheta_k\left(u\ell^{[k+1]} - u\ell^{[k]}\right)$$

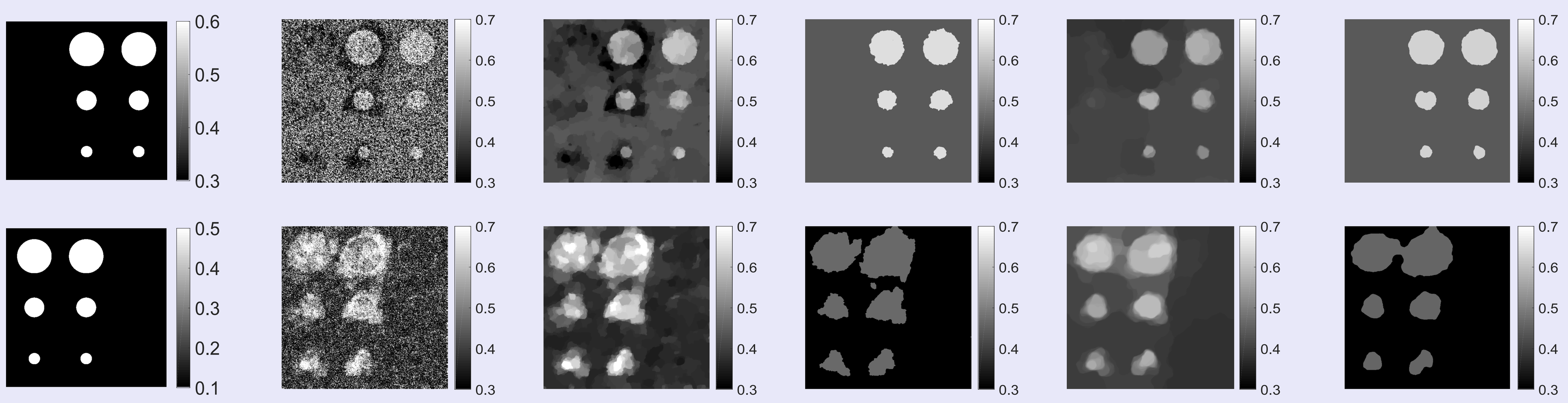
end

Experiments on synthetic textures

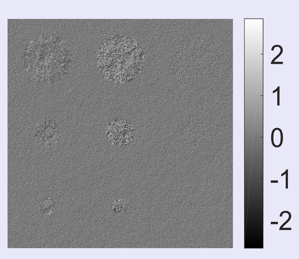
Ground truth | Lin. reg. | Dis. TV | Dis. re-est. | **Joint est.** | **Joint re-est.**

Loc. var.

Loc. reg.



Synth. text.



	Lin. reg.	Dis. TV	Dis. re-est.	**Joint est.**	**Joint re-est.**
$\mathrm{SNR}(v, v_0)$	2.7496	9.9722	8.0758	**10.2854**	8.0241
$\mathrm{SNR}(h, h_0)$	-5.3411	-4.2591	0.14181	-4.1325	**0.24025**

Conclusion

Achieved:

✓ joint estimation of v and h,

✓ lead to reliable texture segmentation,

✓ with efficient implementation provided.

Futur work:

➙ use **DF** into a Mumford-Shah functional,

➙ turn to multifractal texture models.

References

- Jaffard, *Some mathematical results about the multifractal formalism for functions*, 1994
- Wendt *et al.*, *Wavelet leaders and bootstrap for multifractal analysis of images*, 2009
- Chambolle *et al.*, *A first-order primal-dual algorithm for convex problems with applications to imaging*, 2011
- Pustelnik *et al.*, *Combining local regularity estimation and total variation optimization for scale-free texture segmentation*, 2016