Proximal schemes for the estimation of the reproduction number of Covid19:

From convex optimization to Monte Carlo sampling

Groupe de travail Bio-Maths

Institut Denis Poisson

January 25, 2024

Barbara Pascal

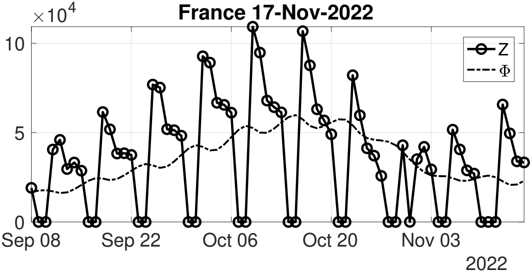
Joint work with P. Abry, N. Pustelnik, S. Roux, R. Gribonval, P. Flandrin;
G. Fort, H. Artigas; Juliana Du

Outline

- Pandemic study: modeling at the service of monitoring
- Reproduction number estimation from minimization of penalized likelihood
- Bayesian framework for credibility interval estimation
- Conclusion & Perspectives

Pandemic study: modeling at the service of monitoring

Counts of daily new infections



data from National Health Agencies collected by Johns Hopkins University

$\Longrightarrow$ number of cases not informative enough: need to capture the **dynamics**

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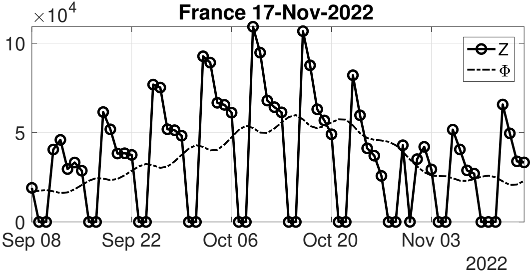


data from National Health Agencies collected by Johns Hopkins University

$\Longrightarrow$ number of cases not informative enough: need to capture the **dynamics**

Design adapted counter measures and evaluate their effectiveness

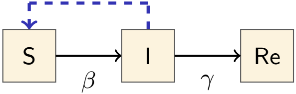
$\longrightarrow$ efficient monitoring tools *epidemiological model,*

$\longrightarrow$ robust to low quality of the data *managing erroneous counts,*

$\longrightarrow$ accompanied by reliable confidence level *credibility intervals.*

Pandemic study: modeling at the service of monitoring

Susceptible-Infected-Recovered (SIR), among *compartmental models*



- ODE: $\frac{\mathrm{dS}_t}{\mathrm{d}t} = -\beta \mathrm{S}_t \mathrm{I}_t$, $\frac{\mathrm{dI}_t}{\mathrm{d}t} = \beta \mathrm{S}_t \mathrm{I}_t - \gamma \mathrm{I}_t$, $\frac{\mathrm{dRe}_t}{\mathrm{d}t} = \gamma \mathrm{I}_t$
- Stochastic model: likelihood maximization to infer β, γ

Pandemic study: modeling at the service of monitoring

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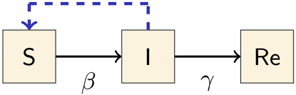

– ODE: $\frac{\mathrm{d}S_t}{\mathrm{d}t} = -\beta S_t I_t$, $\frac{\mathrm{d}I_t}{\mathrm{d}t} = \beta S_t I_t - \gamma I_t$, $\frac{\mathrm{d}Re_t}{\mathrm{d}t} = \gamma I_t$

– Stochastic model: likelihood maximization to infer β, γ

Limitations:

- refinement needed to get socially realistic model
- quadratic increase of the number of parameters
- Bayesian framework: heavy computational burden
- need consolidated and accurate datasets

✗ not adapted to real-time monitoring of Covid19 pandemic

Pandemic study: modeling at the service of monitoring

Reproduction number in Cori model

"averaged number of secondary cases generated by a typical infectious individual"

(Cori et al., 2013, *Am. Journal of Epidemiology*; Liu et al., 2018, *PNAS*)

Pandemic study: modeling at the service of monitoring

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Interpretation: at day t

$\mathrm{R}_t > 1$ the virus propagates at exponential speed,

$\mathrm{R}_t < 1$ the epidemic shrinks with an exponential decay,

$\mathrm{R}_t = 1$ the epidemic is stable.

$\Longrightarrow$ one single indicator accounting for the overall pandemic mechanism

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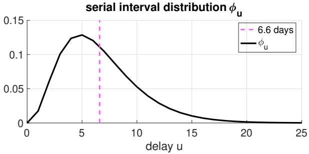
Principle: Z_t new infections at day t

$$\mathbb{E}\left[\mathrm{Z}_t\right] = \mathrm{R}_t \Phi_t, \quad \Phi_t = \sum_{u=1}^{\tau_\Phi} \phi_u \mathrm{Z}_{t-u}$$

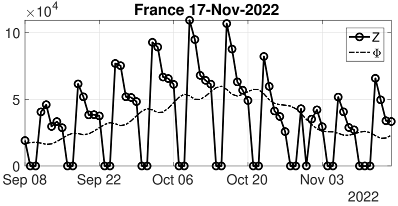


with Φ_t global "infectiousness" in the population

$\{\phi_u\}_{u=1}^{\tau_\Phi}$ distribution of delay between onset of symptoms in primary and secondary cases

Gamma distribution truncated at 25 days, of mean 6.6 days and standard deviation 3.5 days

Pandemic study: modeling at the service of monitoring



Data: daily counts $\mathbf{Z} = (Z_1, \ldots, Z_T)$

Model: Poisson distribution

$$\mathbb{P}(\mathsf{Z}_t | \mathbf{Z}_{t-\tau_\Phi:t-1}, \mathsf{R}_t) = \frac{(\mathsf{R}_t \Phi_t)^{\mathsf{Z}_t} \mathrm{e}^{-\mathsf{R}_t \Phi_t}}{\mathsf{Z}_t!}$$

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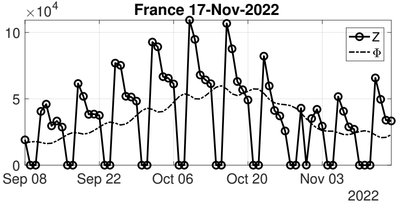

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Maximum Likelihood Estimate (MLE)

$$\begin{aligned}
&\ln\left(\mathbb{P}(Z_t|\mathbf{Z}_{t-\tau_\Phi:t-1}, R_t)\right) \\
&= Z_t \ln(R_t\Phi_t) - R_t\Phi_t - \ln(Z_t!) \\
&\underset{Z_t \gg 1}{\simeq} Z_t \ln(R_t\Phi_t) - R_t\Phi_t - Z_t\ln(Z_t) + Z_t \\
&\underset{\text{(def.)}}{=} -d_{KL}(Z_t|R_t\Phi_t) \quad \text{(Kullback-Leibler)}
\end{aligned}$$

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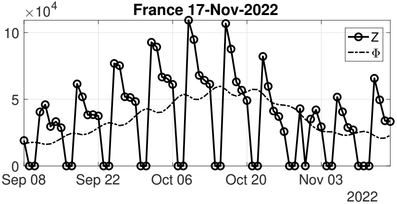

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$$\Longrightarrow \widehat{R}_t^{\text{MLE}} = Z_t / \Phi_t = Z_t / \sum_{u=1}^{\tau_\Phi} \phi_u Z_{t-u}$$

ratio of moving averages

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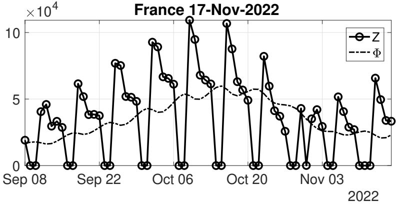

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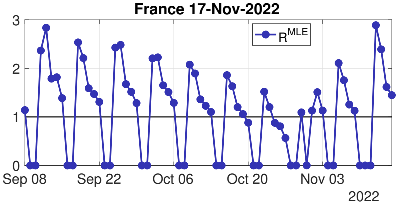


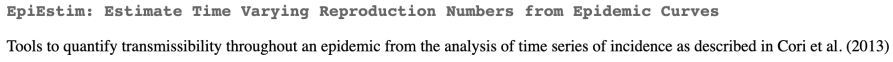
- huge variability along time/ no local trend
- not robust to pseudo-periodicity/ misreported counts

Reproduction number estimation from minimization of penalized likelihood

State-of-the-art: smooth regularization over a temporal window

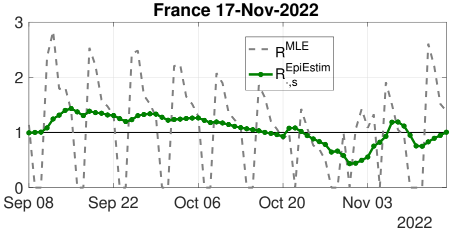
$$\widehat{\mathrm{R}}^{\mathrm{EpiEstim}}_{t,s}, \text{ with } s = 7 \text{ days}$$

(Cori et al., 2013, *Am. Journal of Epidemiology*)



EpiEstim: Estimate Time Varying Reproduction Numbers from Epidemic Curves

Tools to quantify transmissibility throughout an epidemic from the analysis of time series of incidence as described in Cori et al. (2013)



(re-implemented in Matlab following Cori et al., 2013, *Am. Journal of Epidemiology*)

$\Longrightarrow$ smoother than naive MLE but hampered by low quality data and dependent on s

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Solution 1: regularization through nonlinear filtering

$$\widehat{\mathbf{R}}^{\mathrm{PKL}} = \underset{\mathbf{R} \in \mathbb{R}_+^T}{\operatorname{argmin}} \sum_{t=1}^{T} \mathrm{d}_{\mathrm{KL}} \left(\mathrm{Z}_t \,|\mathrm{R}_t \Phi_t \right) + \lambda_{\mathrm{R}} \mathcal{P}(\mathbf{R}) \quad \text{(penalized Kullback-Leibler)}$$

with $\mathcal{P}(\mathbf{R})$ favoring some temporal regularity (Abry et al., 2020, *PLOSOne*)

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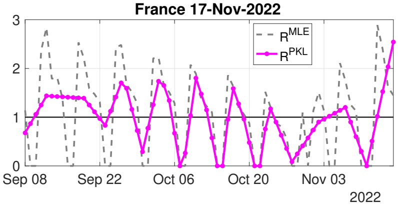
$$\mathcal{P}(\mathbf{R}) = \|\mathbf{D}_2\mathbf{R}\|_1$$

$$(\mathbf{D}_2\mathbf{R})_t = \mathrm{R}_{t+1} - 2\mathrm{R}_t + \mathrm{R}_{t-1}$$

2nd order derivative & ℓ_1-norm

$\Longrightarrow$ piecewise linearity



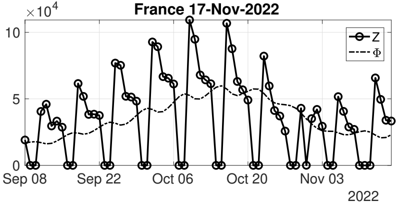
captures global **trend**, more **regular** than MLE, but **pseudo-oscillations**

Reproduction number estimation from minimization of penalized likelihood

New infection counts **Z** are corrupted by

- missing samples,
- non meaningful negative counts,
- retrospected cumulated counts,
- pseudo-seasonality effects.

$\Longrightarrow$ full parametric modeling out of reach



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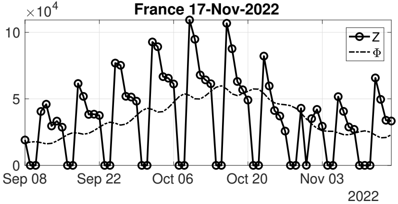

Solution 1': first correct **Z**, then apply penalized Kullback-Leibler on corrected $\mathbf{Z}^{(C)}$

$\Longrightarrow$ two-step procedure not optimal: accumulates correction & regularization biases

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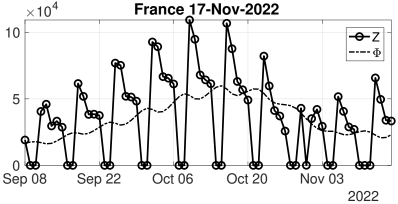

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Solution 2: **one-step** procedure performing jointly

correction of corrupted Z_t & estimation of regularized R_t

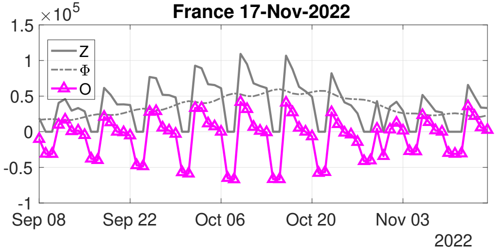
(Pascal et al., 2022, *Trans. Sig. Process.*)

Reproduction number estimation from minimization of penalized likelihood

Extended Cori Model: additional latent variable O_t accounting for misreport

$$Z_t \sim \text{Poiss}\left(R_t\Phi_t + O_t\right), \quad R_t\Phi_t + O_t \geq 0$$

nonzero values of O_t concentrated on specific days (Sundays, day-offs, ...)



Interpretation:

$$\text{Poiss}\left(R_t\Phi_t + O_t\right) \sim \begin{cases} \text{Poiss}\left(R_t\Phi_t\right) + \text{Poiss}\left(O_t\right) & \text{if } O_t \geq 0, \\ \text{Poiss}\left(\alpha_t R_t\Phi_t\right), \ \alpha_t = 1 - \frac{-O_t}{R_t\Phi_t} \in [0,1] & \text{if } O_t < 0. \end{cases}$$

Reproduction number estimation from minimization of penalized likelihood

Data: reported counts $\mathbf{Z} = (Z_1, \ldots, Z_T)$

Model: corrected Poisson $\quad \mathbb{P}(Z_t | \mathbf{Z}_{t-\tau_\Phi:t-1}, R_t, O_t) = \dfrac{(R_t \Phi_t + O_t)^{Z_t} \mathrm{e}^{-(R_t \Phi_t + O_t)}}{Z_t!}$

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Generalized Penalized Kullback-Leibler

$$(\widehat{\mathbf{R}}, \widehat{\mathbf{O}}) \in \underset{(\mathbf{R},\mathbf{O}) \in \mathbb{R}_+^T \times \mathbb{R}^T}{\mathrm{Argmin}} \sum_{t=1}^{T} \mathrm{d}_{\mathrm{KL}}\left(Z_t \,|\, R_t\Phi_t + O_t\right) + \lambda_{\mathrm{R}} \|\mathbf{D}_2 \mathbf{R}\|_1 + \iota_{\geq 0}(\mathbf{R}) + \lambda_{\mathrm{O}} \|\mathbf{O}\|_1$$

$\Longrightarrow$ estimates piecewise linear, non-negative R_t and sparse O_t

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properties of the objective function:

- sum of convex functions composed with linear operators $\Longrightarrow$ globally convex;
- feasible domain: $(\forall t, R_t \geq 0)$ & (if $Z_t > 0, R_t\Phi_t + O_t > 0$, else $R_t\Phi_t + O_t \geq 0$);
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Theorem (Pascal et al., 2022, *Trans. Sig. Process.*)

+ The minimization problem has at least one solution $(\widehat{\mathbf{R}}, \widehat{\mathbf{O}})$.
+ The estimated time-varying Poisson intensity $\widehat{p}_t = \widehat{R}_t\Phi_t + \widehat{O}_t$ is unique.

Reproduction number estimation from minimization of penalized likelihood

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- each term of the functional is convex;
- ℓ_1-norm and indicative function $\Longrightarrow$ nonsmooth; ✗ gradient descent
- gradient of $\mathrm{p}_t \mapsto \mathrm{d}_{\mathrm{KL}}\left(\mathrm{Z}_t \,|\, \mathrm{p}_t\right)$ is not Lipschitzian; ✗ forward-backward
- linear operator $\mathbf{D}_2 \Longrightarrow$ no explicit form for $\mathrm{prox}_{\|\mathbf{D}_2\cdot\|_1}$ ♣ need splitting

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$$\Longleftrightarrow \underset{(\mathbf{R},\mathbf{O})\in\mathbb{R}_+^T\times\mathbb{R}^T}{\text{minimize}} \quad f(\mathbf{R},\mathbf{O}|\mathbf{Z}) + h(\mathbf{A}(\mathbf{R},\mathbf{O})), \quad \mathbf{A} \text{ linear; } f, h \text{ proximable}$$

$$\mathbf{A}(\mathbf{R},\mathbf{O}) = (\lambda_{\mathrm{R}}\mathbf{D}_2\mathbf{R}, \mathbf{R}, \lambda_{\mathrm{O}}\mathbf{O})\,; \quad h\left(\mathbf{Q}_1,\mathbf{Q}_2,\mathbf{Q}_3\right) = \|\mathbf{Q}_1\|_1 + \iota_{\geq 0}(\mathbf{Q}_2) + \|\mathbf{Q}_3\|_1$$

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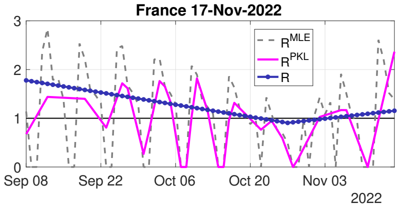
Primal-dual algorithm (Chambolle et al., 2011, *Int. Conf. Comput. Vis.*)

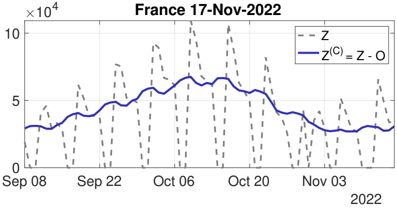
```
for $k = 1, 2 \ldots$ do
    $\mathbf{Q}^{[k+1]} = \mathrm{prox}_{\sigma h^*}(\mathbf{Q}^{[k]} + \sigma\mathbf{A}(\overline{\mathbf{R}}^{[k]}, \overline{\mathbf{O}}^{[k]}))$    dual
    $(\mathbf{R}^{[k+1]}, \mathbf{O}^{[k+1]}) = \mathrm{prox}_{\tau f(\cdot|\mathbf{Z})}((\mathbf{R}^{[k+1]}, \mathbf{O}^{[k+1]}) - \tau\mathbf{A}^*\mathbf{Q}^{[k+1]})$    primal
    $(\overline{\mathbf{R}}^{[k+1]}, \overline{\mathbf{O}}^{[k+1]}) = 2(\mathbf{R}^{[k+1]}, \mathbf{O}^{[k+1]}) - (\mathbf{R}^{[k]}, \mathbf{O}^{[k]})$    auxiliary
```

Reproduction number estimation from minimization of penalized likelihood

Reproduction number $\widehat{\mathbf{R}}$



Corrected infection counts $\mathbf{Z}^{(C)}$



$\Longrightarrow$ no more pseudo-seasonality, local trends well captured, smooth behavior

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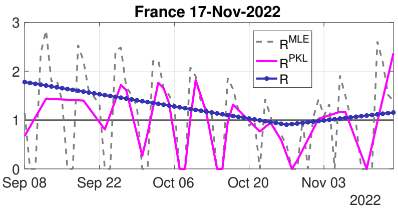

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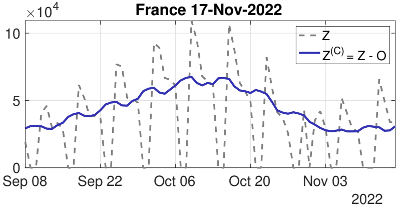

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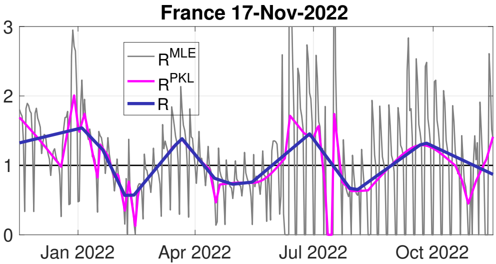

fast numerical scheme: 15 to 30 sec for 70 days to 1 year

Reproduction number estimation from minimization of penalized likelihood

New infection counts per county: $\mathbf{Z} = \left\{ \mathsf{Z}_t^{(d)},\ d \in [1, D],\ t \in [1,\ T] \right\}$

$\Rightarrow$ multivariate time-varying reproduction number $\mathsf{R}_t^{(d)}$

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Multivariate extended penalized Kullback-Leibler

$$\left(\widehat{\mathbf{R}}, \widehat{\mathbf{O}}\right) = \underset{(\mathbf{R},\mathbf{O}) \in \mathbb{R}_+^{D\times T} \times \mathbb{R}^{D\times T}}{\operatorname{argmin}} \sum_{d=1}^{D} \sum_{t=1}^{T} \mathsf{d}_{\mathsf{KL}} \left(\mathsf{Z}_t^{(d)} \middle| \mathsf{R}_t^{(d)} \Phi_t^{(d)} + \mathsf{O}_t^{(d)} \right)$$
$$+ \lambda_{\mathsf{R}} \|\mathbf{D}_2 \mathbf{R}\|_1 + \iota_{\geq 0}(\mathbf{R}) + \lambda_{\text{space}} \|\mathbf{G}\mathbf{R}\|_1 + \lambda_{\mathsf{O}} \|\mathbf{O}\|_1$$

$\Longrightarrow$ $\|\mathbf{G}\mathbf{R}\|_1$ favors **piecewise constancy** in space

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Graph Total Variation

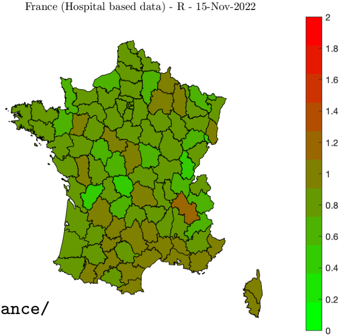
$$\|\mathbf{G}\mathbf{R}\|_1 = \sum_{t=1}^{T} \sum_{d_1 \sim d_2} \left| \mathsf{R}_t^{(d_1)} - \mathsf{R}_t^{(d_2)} \right|$$

sum over neighboring counties

here: $d_1 \sim d_2 \Leftrightarrow$ share terrestrial border

$$\widetilde{\mathbf{A}}(\mathbf{R}, \mathbf{O}) = (\lambda_{\mathsf{R}} \mathbf{D}_2 \mathbf{R}, \mathbf{R}, \lambda_{\text{space}} \mathbf{G}\mathbf{R}, \lambda_{\mathsf{O}} \mathbf{O})$$

`http://barthes.enssib.fr/coronavirus/cartes/RFrance/`



Bayesian framework for credibility interval estimation

Pointwise estimate of parameter $\boldsymbol{\theta} = (\mathbf{R}, \mathbf{O})$ from observations **Z**

$$\underset{\boldsymbol{\theta} \in \mathbb{R}_+^T \times \mathbb{R}^T}{\text{minimize}} \quad f(\boldsymbol{\theta}|\mathbf{Z}) + h(\mathbf{A}\boldsymbol{\theta})$$

(Pascal et al., 2022, *Trans. Sig. Process.*)

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Q: what is the value of R today? **A:** solve the minimization problem and output $\widehat{\mathsf{R}}_T$.

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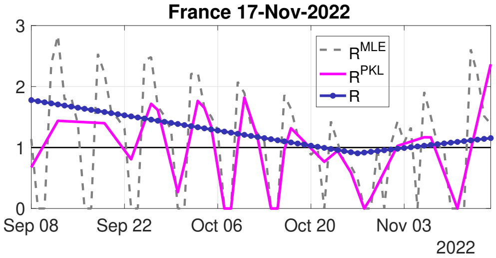

$\widehat{\mathrm{R}}_T = 1.2955$

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Bayesian reformulation: interpret $(\widehat{\mathbf{R}}, \widehat{\mathbf{O}})$ as the MAP of

$$\pi(\boldsymbol{\theta}) \propto \exp(-f(\boldsymbol{\theta}|\mathbf{Z}) - h(\mathbf{A}\boldsymbol{\theta}))$$

- $\exp(-f(\boldsymbol{\theta}|\mathbf{Z})) \sim$ likelihood of the observation
- $\exp(-h(\mathbf{A}\boldsymbol{\theta})) \sim$ prior on the parameter of interest

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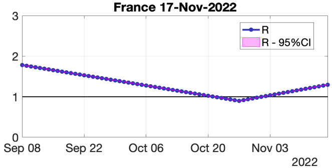
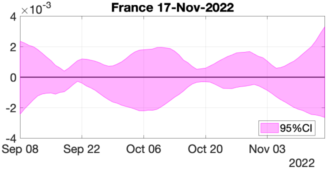
$\Longrightarrow$ instead of focusing on $\widehat{\mathrm{R}}_t$, the **pointwise** MAP, probe π to get

$\mathrm{R}_t \in [\underline{\mathrm{R}}_t, \overline{\mathrm{R}}_t]$ with 95% probability, i.e., **credibility interval** estimates





$\widehat{\mathrm{R}}_T \in [1.2987, 1.3047]$

Bayesian framework for credibility interval estimation

Log-likelihood from Poisson model $\mathcal{D} = \{\boldsymbol{\theta} \,|\, \forall t,\ \ \mathsf{R}_t \Phi_t + \mathsf{O}_t \geq 0,\ \mathsf{R}_t \geq 0\}$

$$f(\boldsymbol{\theta} \,|\mathbf{Z}) := \begin{cases} -\sum_{t=1}^{T} (\mathsf{Z}_t \ln(\mathsf{R}_t \Phi_t + \mathsf{O}_t) - (\mathsf{R}_t \Phi_t + \mathsf{O}_t) + \mathcal{C}(\mathsf{Z}_t)) & \text{if } \boldsymbol{\theta} \in \mathcal{D}, \\ \infty & \text{otherwise,} \end{cases}$$

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$$\Rightarrow g(\boldsymbol{\theta}) = \lambda_\mathsf{R}\|\mathbf{D}_2\mathbf{R}\|_1 + \lambda_\mathsf{O}\|\mathbf{O}\|_1, \quad \mathbf{D}_2 = \underbrace{\frac{1}{\sqrt{6}}\begin{bmatrix} 1 & -2 & 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & -2 & 1 & 0 & \ldots & 0 \\ \ldots & & & & & & \ldots \\ 0 & \ldots & & & 1 & -2 & 1 \end{bmatrix}}_{\text{Laplacian}}$$

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Posterior distribution of unknown parameters $\boldsymbol{\theta} = (\mathbf{R}, \mathbf{O})$

$$\pi(\boldsymbol{\theta}) \propto \exp\left(-f(\boldsymbol{\theta}) - g(\boldsymbol{\theta})\right) \mathbb{1}_{\mathcal{D}}(\boldsymbol{\theta})$$

- f, g convex
- f smooth, g nonsmooth

Markov Chain Monte Carlo sampling

Purpose: sampling the random variable $\boldsymbol{\theta} = (\mathbf{R}, \mathbf{O}) \in \mathbb{R}^{2T}$ according to the posterior[†]

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- at convergence, i.e., as $n \to \infty$, $\boldsymbol{\theta}^n \sim \pi$,

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Simple and very general approach: *Hastings-Metropolis random walk*

(i) propose a random move according to

$$\boldsymbol{\theta}^{n+\frac{1}{2}} = \boldsymbol{\theta}^n + \sqrt{2\gamma}\Gamma\xi^{n+1}, \quad \xi^{n+1} \sim \mathcal{N}_{2T}(0, \mathrm{I})$$

with γ positive step size, $\Gamma \in \mathbb{R}^{2T \times 2T}$

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(ii) accept: $\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta}^{n+\frac{1}{2}}$, with probability $1 \wedge \dfrac{\pi(\boldsymbol{\theta}^{n+\frac{1}{2}})}{\pi(\boldsymbol{\theta}^n)}$, or reject: $\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta}^n$

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Metropolis Adjusted Langevin Algorithm (MALA)

Langevin dynamics: $\boldsymbol{\theta}^{n+\frac{1}{2}} = \mu(\boldsymbol{\theta}^{n}) + \sqrt{2\gamma}\xi^{n+1}$, (Kent, 1978, *Adv Appl Probab*)

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Case 1: $g = 0$ and $-\ln\pi = f$ is smooth (Roberts & Tweedie, 1996, *Bernoulli*)

$$\mu(\boldsymbol{\theta}) = \boldsymbol{\theta} - \gamma\Gamma\Gamma^{\top}\nabla f(\boldsymbol{\theta}) = \boldsymbol{\theta} + \gamma\Gamma\Gamma^{\top}\nabla\ln\pi(\boldsymbol{\theta})$$

$\Longrightarrow$ move towards areas of higher probability

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Case 2: $-\ln\pi = f + g$ is nonsmooth

$$\mu(\boldsymbol{\theta}) = \mathrm{prox}_{\gamma g}^{\Gamma\Gamma^\top}(\boldsymbol{\theta} - \gamma\Gamma\Gamma^\top\nabla f(\boldsymbol{\theta}))$$

combining *Langevin* and *proximal*[†] approaches

[†] $\mathrm{prox}_{\gamma g}^{\Gamma\Gamma^\top}(y) = \underset{x\in\mathbb{R}^d}{\mathrm{argmin}}\left(\frac{1}{2}\|x-y\|^2_{\Gamma\Gamma^\top} + \gamma g(x)\right)$: preconditioned proximity operator of g

Proximal-Gradient dual sampler `PGdual`

Posterior density of $\boldsymbol{\theta} = (\mathbf{R}, \mathbf{O})$: $\quad \pi(\boldsymbol{\theta}) \propto \exp\left(-f(\boldsymbol{\theta}) - g(\boldsymbol{\theta})\right) \mathbb{1}_{\mathcal{D}}(\boldsymbol{\theta})$

- smooth negative log-likelihood

$$\text{if } \boldsymbol{\theta} \in \mathcal{D}, \quad f(\boldsymbol{\theta}) = -\sum_{t=1}^{T} \left(\mathsf{Z}_t \ln \mathsf{p}_t(\boldsymbol{\theta}) - \mathsf{p}_t(\boldsymbol{\theta})\right), \quad \mathsf{p}_t(\boldsymbol{\theta}) = \mathsf{R}_t(\Phi \mathsf{Z})_t + \mathsf{O}_t$$

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Case 3: $-\ln \pi = f + h(\mathbf{A}\cdot)$ (Fort et al., 2022, *preprint*)

closed-form expression of $\mathrm{prox}_{\gamma h}$ but not of $\mathrm{prox}_{\gamma h(\mathbf{A}\cdot)}$

1) extend $\mathbf{A}$ into **invertible** $\overline{\mathbf{A}}$, and h in $\bar{h}$ such that $\bar{h}(\overline{\mathbf{A}}\boldsymbol{\theta}) = h(\mathbf{A}\boldsymbol{\theta})$

2) reason on the **dual** variable $\tilde{\boldsymbol{\theta}} = \overline{\mathbf{A}}\boldsymbol{\theta}$

Proximal-Gradient dual sampler `PGdual`

Langevin: drift toward higher probability regions

$$\operatorname*{argmax}_{\boldsymbol{\theta}\in\mathbb{R}^{2T}} \ln \pi(\boldsymbol{\theta}) = \operatorname*{argmin}_{\boldsymbol{\theta}\in\mathbb{R}^{2T}} f(\boldsymbol{\theta}) + \bar{h}(\overline{\mathbf{A}}\boldsymbol{\theta}) = \mathbf{A}^{-1}\operatorname*{argmin}_{\tilde{\boldsymbol{\theta}}\in\mathbb{R}^{2T}} f(\overline{\mathbf{A}}^{-1}\tilde{\boldsymbol{\theta}}) + \bar{h}(\tilde{\boldsymbol{\theta}})$$

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$$\Longrightarrow \quad \mu(\boldsymbol{\theta}) = \underbrace{\overline{\mathbf{A}}^{-1}}_{\text{back to } \boldsymbol{\theta}} \ \underbrace{\operatorname{prox}_{\gamma\bar{h}}\left(\overline{\mathbf{A}}\boldsymbol{\theta} - \gamma\overline{\mathbf{A}}^{-\top}\nabla f(\boldsymbol{\theta})\right)}_{\text{proximal-gradient on } \tilde{\boldsymbol{\theta}}}$$

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$$\underset{\boldsymbol{\theta}\in\mathbb{R}^{2T}}{\operatorname{argmax}} \ln \pi(\boldsymbol{\theta}) = \underset{\boldsymbol{\theta}\in\mathbb{R}^{2T}}{\operatorname{argmin}}\ f(\boldsymbol{\theta}) + \bar{h}(\overline{\mathbf{A}}\boldsymbol{\theta}) = \mathbf{A}^{-1} \underset{\tilde{\boldsymbol{\theta}}\in\mathbb{R}^{2T}}{\operatorname{argmin}}\ f(\overline{\mathbf{A}}^{-1}\tilde{\boldsymbol{\theta}}) + \bar{h}(\tilde{\boldsymbol{\theta}})$$

$$\Longrightarrow \quad \mu(\boldsymbol{\theta}) = \underbrace{\overline{\mathbf{A}}^{-1}}_{\text{back to } \boldsymbol{\theta}} \underbrace{\operatorname{prox}_{\gamma\bar{h}}\left(\overline{\mathbf{A}}\boldsymbol{\theta} - \gamma\overline{\mathbf{A}}^{-\top}\nabla f(\boldsymbol{\theta})\right)}_{\text{proximal-gradient on } \tilde{\boldsymbol{\theta}}}$$

Two strategies to extend $\mathbf{A} = \begin{pmatrix} \mathbf{D}_2 & 0 \\ 0 & \mathbf{I} \end{pmatrix} \in \mathbb{R}^{(2T-1)\times 2T}$ into $\overline{\mathbf{A}} = \begin{pmatrix} \overline{\mathbf{D}} & 0 \\ 0 & \mathbf{I} \end{pmatrix} \in \mathbb{R}^{2T\times 2T}$:

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`Invert`

$$\overline{\mathbf{D}}_2 := \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -2/\sqrt{5} & 1/\sqrt{5} & 0 & \cdots & 0 \\ & \mathbf{D}_2 & & & \end{bmatrix}$$

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`Ortho`

$$\overline{\mathbf{D}}_o := \begin{bmatrix} v_1^\top \\ v_2^\top \\ \mathbf{D}_2 \end{bmatrix} \qquad \begin{array}{c} v_1, v_2 \in \mathbb{R}^{2T} \\ v_1 \perp v_2,\ v_1, v_2 \in (\mathbf{D}_2^\top)^\perp \end{array}$$

Proximal-Gradient dual sampler `PGdual`

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Proposed `PGdual` **drift terms** on $\boldsymbol{\theta} = (\mathbf{R}, \mathbf{O})$:

reproduction numbers $\mu_{\mathrm{R}}(\boldsymbol{\theta}) = \overline{\mathbf{D}}^{-1} \operatorname{prox}_{\gamma_{\mathrm{R}}\lambda_{\mathrm{R}}\|(\cdot)_{3:T}\|_1}\left(\overline{\mathbf{D}}\,\mathbf{R} - \gamma_{\mathrm{R}}\overline{\mathbf{D}}^{-\top}\nabla_{\mathrm{R}} f(\boldsymbol{\theta})\right)$

outliers $\mu_{\mathrm{O}}(\boldsymbol{\theta}) = \operatorname{prox}_{\gamma_{\mathrm{O}}\lambda_{\mathrm{O}}\|\cdot\|_1}\left(\mathbf{O} - \gamma_{\mathrm{O}}\nabla_{\mathrm{O}} f(\boldsymbol{\theta})\right)$

Markov Chain Monte Carlo sampling scheme

```
Data: $\overline{\mathbf{D}} = \overline{\mathbf{D}}_2$ (Invert) or $\overline{\mathbf{D}} = \overline{\mathbf{D}}_o$ (Ortho)
      $\gamma_\mathrm{R}, \gamma_\mathrm{O} > 0$, $N_\mathrm{max} \in \mathbb{N}_\star$, $\boldsymbol{\theta}^0 = (\mathbf{R}^0, \mathbf{O}^0) \in \mathcal{D}$
Result: A $\mathcal{D}$-valued sequence $\{\boldsymbol{\theta}^n = (\mathbf{R}^n, \mathbf{O}^n), n \in 0, \ldots, N_\mathrm{max}\}$
for $n = 0, \ldots, N_\mathrm{max} - 1$ do
    Sample $\xi_\mathrm{R}^{n+1} \sim \mathcal{N}_T(0, \mathrm{I})$ and $\xi_\mathrm{O}^{n+1} \sim \mathcal{N}_T(0, \mathrm{I})$;
    Set $\mathbf{R}^{n+\frac{1}{2}} = \mu_\mathrm{R}(\boldsymbol{\theta}^n) + \sqrt{2\gamma_\mathrm{R}}\,\overline{\mathbf{D}}^{-1}\overline{\mathbf{D}}^{-\top}\,\xi_\mathrm{R}^{n+1}$ ;
        $\mathbf{O}^{n+\frac{1}{2}} = \mu_\mathrm{O}(\boldsymbol{\theta}^n) + \sqrt{2\gamma_\mathrm{O}}\,\xi_\mathrm{O}^{n+1}$;
        $\boldsymbol{\theta}^{n+\frac{1}{2}} = (\mathbf{R}^{n+\frac{1}{2}}, \mathbf{O}^{n+\frac{1}{2}})$ ;
    Set $\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta}^{n+\frac{1}{2}}$ with probability

        $1 \wedge \frac{\pi(\boldsymbol{\theta}^{n+\frac{1}{2}})}{\pi(\boldsymbol{\theta}^n)} \frac{q_\mathrm{R}(\boldsymbol{\theta}^{n+\frac{1}{2}}, \boldsymbol{\theta}_\mathrm{R}^n)}{q_\mathrm{R}(\boldsymbol{\theta}^n, \boldsymbol{\theta}_\mathrm{R}^{n+\frac{1}{2}})} \frac{q_\mathrm{O}(\boldsymbol{\theta}^{n+\frac{1}{2}}, \boldsymbol{\theta}_\mathrm{O}^n)}{q_\mathrm{O}(\boldsymbol{\theta}^n, \boldsymbol{\theta}_\mathrm{O}^{n+\frac{1}{2}})}$,

        $q_\mathrm{R/O}$ :  Gaussian kernel stemming from nonsymmetric proposal

     and $\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta}^n$ otherwise.
```

Algorithm 1: Proximal-Gradient dual: `PGdual Invert` and `PGdual Ortho`

Comparison of MCMC sampling schemes

Gaussian proposal: $\boldsymbol{\theta}^{n+\frac{1}{2}} = \mu(\boldsymbol{\theta}^n) + \sqrt{2\gamma}\Gamma\xi^{n+1}$

- random walks: $\mu(\boldsymbol{\theta}) = \boldsymbol{\theta}$

  `RW`: $\Gamma = \mathsf{I}$; `RW Invert`: $\Gamma = \overline{\mathbf{D}_2}^{-1}\overline{\mathbf{D}_2}^{-\top}$; `RW Ortho`: $\Gamma = \overline{\mathbf{D}_o}^{-1}\overline{\mathbf{D}_o}^{-\top}$

- Proximal-Gradient dual: $\mu_{\mathrm{R}}(\boldsymbol{\theta})$, $\mu_{\mathrm{O}}(\boldsymbol{\theta})$, $\Gamma = \overline{\mathbf{D}}^{-1}\overline{\mathbf{D}}^{-\top}$

  `PGdual Invert`: $\overline{\mathbf{D}} = \overline{\mathbf{D}}_2$; `PGdual Ortho`: $\overline{\mathbf{D}} = \overline{\mathbf{D}}_o$

Practical settings: $N_{\max} = 10^7$ iterations, 15 independent runs

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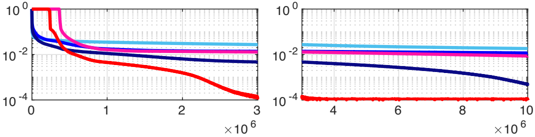
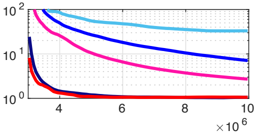
log-density

burn in   post burn in



$(\ln \pi(\boldsymbol{\theta}^n) - \max \ln \pi) \,/\, \left(\ln \pi(\boldsymbol{\theta}^0) - \max \ln \pi\right)$

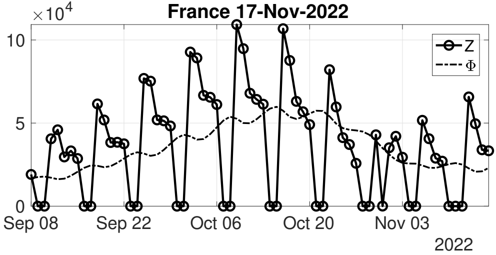
Gelman-Rubin

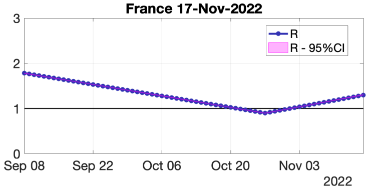


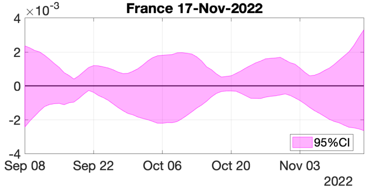
ANOVA-type criterion

`PGdual` credibility interval estimation of the reproduction number

Sanitary situation in France

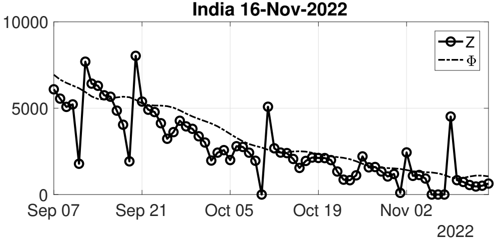


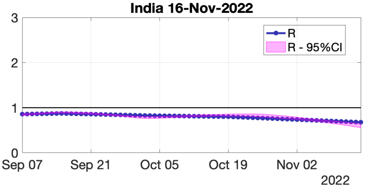


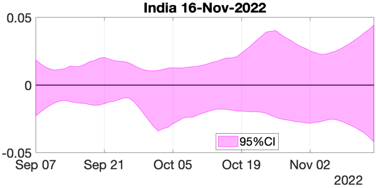


`PGdual` credibility interval estimation of the reproduction number

Worldwide Covid19 monitoring







`PGdual` credibility interval estimation of the reproduction number

Why not United Kingdom?

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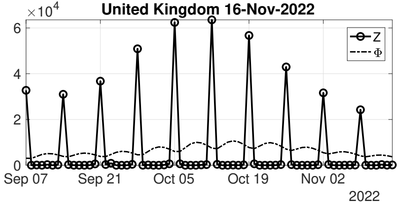
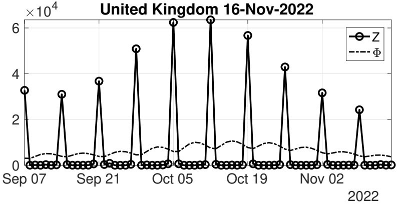


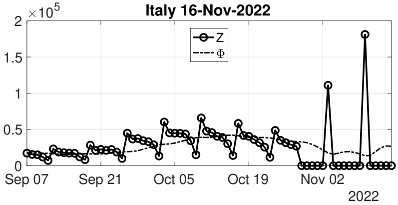
rate of erroneous counts: $6/7$!

`PGdual` credibility interval estimation of the reproduction number

Why not United Kingdom?

And Italy?





rate of erroneous counts: $6/7$!

seems to adopt the same reporting rate ...

$\Longrightarrow$ call for new tools, robust to very scarce data

Conclusion

✓ Extended Cori model handling erroneous reported counts via a latent variable

$$\mathsf{Z}_t | \mathbf{Z}_{t-\tau_\Phi:t-1}, \mathsf{R}_t, \mathsf{O}_t \sim \mathsf{Poiss}(\mathsf{R}_t \Phi_t + \mathsf{O}_t)$$

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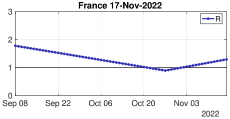
$$\underset{(\mathbf{R},\mathbf{O})\in\mathbb{R}_+^T\times\mathbb{R}^T}{\text{minimize}} \sum_{t=1}^{T} \mathrm{d}_{\mathrm{KL}}\left(\mathsf{Z}_t \,|\, \mathsf{R}_t\Phi_t + \mathsf{O}_t\right) + \lambda_{\mathsf{R}}\|\mathbf{D}_2\mathbf{R}\|_1 + \iota_{\geq 0}(\mathbf{R}) + \lambda_{\mathsf{O}}\|\mathbf{O}\|_1$$



$\widehat{\mathsf{R}}_T = 1.1959$

(Pascal et al., 2022, *Trans. Sig. Process.*;)

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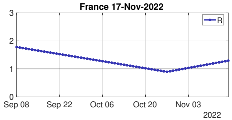
✓ Bayesian credibility interval estimates via proximal Langevin MCMC samplers



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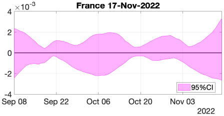

$\widehat{\mathrm{R}}_T \in [1.1978, 1.2016]$

(Pascal et al., 2022, *Trans. Sig. Process.*; Fort et al., 2022, *arXiv:2203.09142*)

Perspectives

⟶ Avoid mixing errors O_t with the pandemic mechanism $\mathsf{R}_t\Phi_t$: anomaly models

$$\mathsf{Z}_t|\mathbf{Z}_{t-\tau_\Phi:t-1}, \mathsf{R}_t, \mathsf{O}_t \sim \mathrm{Poiss}((1-\mathsf{e}_t)\mathsf{R}_t\Phi_t + \mathsf{e}_t\mathsf{O}_t), \quad \mathsf{e}_t \in \{0,1\}$$

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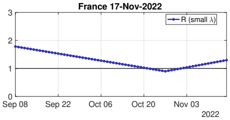
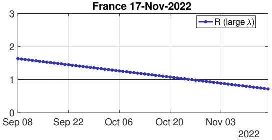
$\longrightarrow$ Avoid mixing errors O_t with the pandemic mechanism $R_t\Phi_t$: anomaly models

$$Z_t | \mathbf{Z}_{t-\tau_\Phi:t-1}, R_t, O_t \sim \text{Poiss}((1-e_t)R_t\Phi_t + e_t O_t), \quad e_t \in \{0,1\}$$

$\longrightarrow$ Selection of regularization parameters λ_R, λ_O

$$\underset{(\mathbf{R},\mathbf{O})\in\mathbb{R}_+^T\times\mathbb{R}^T}{\text{minimize}} \sum_{t=1}^{T} d_{KL}\left(Z_t \,|\, R_t\Phi_t + O_t\right) + \lambda_R\|\mathbf{D}_2\mathbf{R}\|_1 + \iota_{\geq 0}(\mathbf{R}) + \lambda_O\|\mathbf{O}\|_1$$





Juliana Du PhD thesis

$\longrightarrow$ Synthetic data