Proximal schemes for the estimation of the reproduction number of Covid19:

From convex optimization to Monte Carlo sampling

Signal and image processing seminar

Laboratory for Integration from Material to System

October 19th 2023

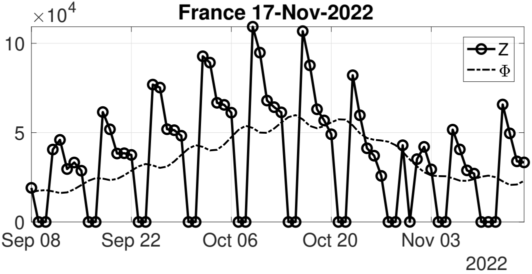
Barbara Pascal

Joint work with P. Abry, N. Pustelnik, S. Roux, R. Gribonval, P. Flandrin; G. Fort, H. Artigas; Juliana Du

Outline

- Pandemic study: modeling at the service of monitoring
- Reproduction number estimation from minimization of penalized likelihood
- Bayesian framework for credibility interval estimation
- Conclusion & Perspectives

Counts of daily new infections



data from National Health Agencies collected by Johns Hopkins University

$\Longrightarrow$ number of cases not informative enough: need to capture the **dynamics**

Pandemic study: modeling at the service of monitoring

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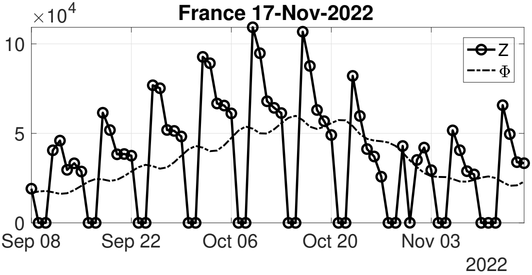

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$\Longrightarrow$ number of cases not informative enough: need to capture the **dynamics**

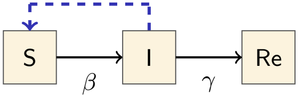
Design adapted counter measures and evaluate their effectiveness

$\longrightarrow$ efficient monitoring tools *epidemiological model,*

$\longrightarrow$ robust to low quality of the data *managing erroneous counts,*

$\longrightarrow$ accompanied by reliable confidence level *credibility intervals.*

Susceptible-Infected-Recovered (SIR), among *compartmental models*



– ODE: $\frac{\mathrm{d}S_t}{\mathrm{d}t} = -\beta S_t I_t$, $\frac{\mathrm{d}I_t}{\mathrm{d}t} = \beta S_t I_t - \gamma I_t$, $\frac{\mathrm{d}Re_t}{\mathrm{d}t} = \gamma I_t$

– Stochastic model: likelihood maximization to infer β, γ

Pandemic study: modeling at the service of monitoring

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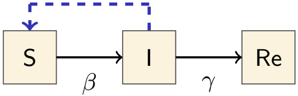

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– Stochastic model: likelihood maximization to infer β, γ

Limitations:

- refinement needed to get socially realistic model
- quadratic increase of the number of parameters
- Bayesian framework: heavy computational burden
- need consolidated and accurate datasets

✗ not adapted to real-time monitoring of Covid19 pandemic

Reproduction number in Cori model

"averaged number of secondary cases generated by a typical infectious individual"

(Cori et al., 2013, *Am. Journal of Epidemiology*; Liu et al., 2018, *PNAS*)

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Interpretation: at day t

$\mathrm{R}_t > 1$ the virus propagates at exponential speed,

$\mathrm{R}_t < 1$ the epidemic shrinks with an exponential decay,

$\mathrm{R}_t = 1$ the epidemic is stable.

$\Longrightarrow$ one single indicator accounting for the overall pandemic mechanism

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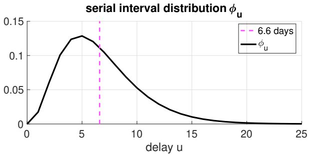
Principle: Z_t new infections at day t

$$\mathbb{E}\left[\mathrm{Z}_t\right] = \mathrm{R}_t \Phi_t, \quad \Phi_t = \sum_{u=1}^{\tau_\Phi} \phi_u \mathrm{Z}_{t-u}$$



with Φ_t global "infectiousness" in the population

$\{\phi_u\}_{u=1}^{\tau_\Phi}$ distribution of delay between onset of symptoms in primary and secondary cases

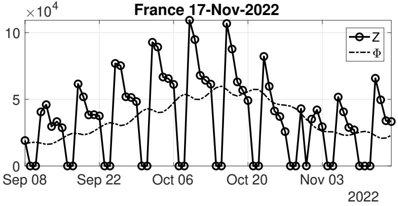
Gamma distribution truncated at 25 days, of mean 6.6 days and standard deviation 3.5 days

Pandemic study: modeling at the service of monitoring

Data: daily counts $\mathbf{Z} = (Z_1, \ldots, Z_T)$

Model: Poisson distribution

$$\mathbb{P}(Z_t | \mathbf{Z}_{t-\tau_\Phi:t-1}, R_t) = \frac{(R_t \Phi_t)^{Z_t} e^{-R_t \Phi_t}}{Z_t!}$$



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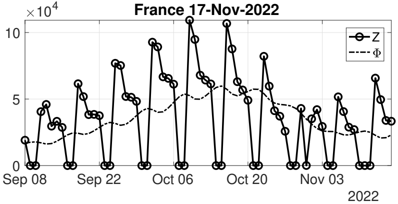

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Maximum Likelihood Estimate (MLE)

$$\begin{aligned} &\ln\left(\mathbb{P}(Z_t | \mathbf{Z}_{t-\tau_\Phi:t-1}, R_t)\right) \\ &= Z_t \ln(R_t \Phi_t) - R_t \Phi_t - \ln(Z_t!) \\ &\underset{Z_t \gg 1}{\simeq} Z_t \ln(R_t \Phi_t) - R_t \Phi_t - Z_t \ln(Z_t) + Z_t \\ &\underset{\text{(def.)}}{=} -d_{KL}(Z_t | R_t \Phi_t) \quad \text{(Kullback-Leibler)} \end{aligned}$$

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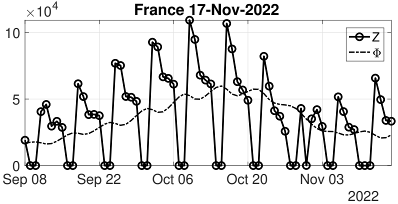

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ratio of moving averages

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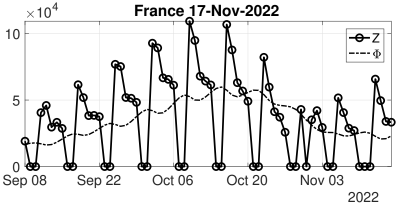

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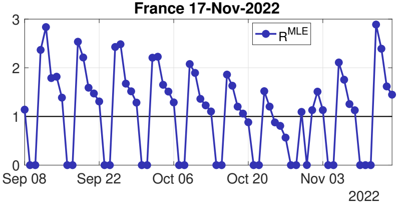


– huge variability along time/ no local trend

– not robust to pseudo-periodicity/ misreported counts

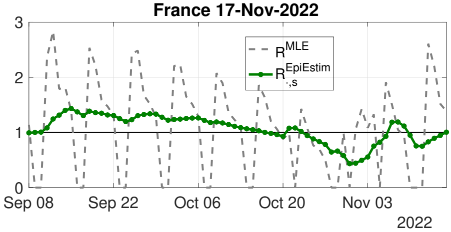
State-of-the-art: smooth regularization over a temporal window

$$\widehat{\mathrm{R}}^{\mathrm{EpiEstim}}_{t,s}, \text{ with } s = 7 \text{ days}$$

(Cori et al., 2013, *Am. Journal of Epidemiology*)

EpiEstim: Estimate Time Varying Reproduction Numbers from Epidemic Curves

Tools to quantify transmissibility throughout an epidemic from the analysis of time series of incidence as described in Cori et al. (2013)



(re-implemented in Matlab following Cori et al., 2013, *Am. Journal of Epidemiology*)

$\Longrightarrow$ smoother than naive MLE but hampered by low quality data and dependent on s

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Solution 1: regularization through nonlinear filtering

$$\widehat{\mathbf{R}}^{\mathrm{PKL}} = \underset{\mathbf{R} \in \mathbb{R}_+^T}{\operatorname{argmin}} \sum_{t=1}^{T} \mathrm{d}_{\mathrm{KL}}\left(\mathrm{Z}_t \,|\mathrm{R}_t \Phi_t\right) + \lambda_{\mathrm{R}} \mathcal{P}(\mathbf{R}) \quad \text{(penalized Kullback-Leibler)}$$

with $\mathcal{P}(\mathbf{R})$ favoring some temporal regularity (Abry et al., 2020, *PLOSOne*)

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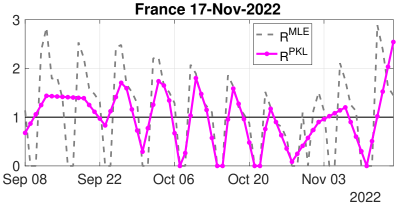
$$\mathcal{P}(\mathbf{R}) = \|\mathbf{D}_2\mathbf{R}\|_1$$

$$(\mathbf{D}_2\mathbf{R})_t = \mathrm{R}_{t+1} - 2\mathrm{R}_t + \mathrm{R}_{t-1}$$

2nd order derivative & ℓ_1-norm

$\Longrightarrow$ piecewise linearity

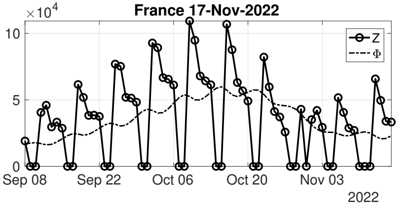


captures global **trend**, more **regular** than MLE, but **pseudo-oscillations**

New infection counts **Z** are corrupted by

- missing samples,
- non meaningful negative counts,
- retrospected cumulated counts,
- pseudo-seasonality effects.

$\Longrightarrow$ full parametric modeling out of reach



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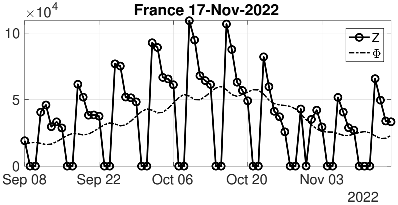

Solution 1': first correct $\mathbf{Z}$, then apply penalized Kullback-Leibler on corrected $\mathbf{Z}^{(C)}$

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(Abry et al., 2020, *Eng. Med. Biol. Conf.*)

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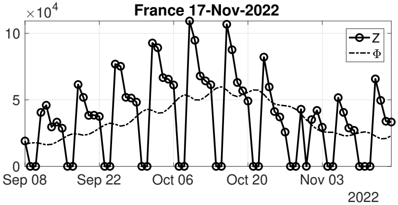

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Solution 2: **one-step** procedure performing jointly

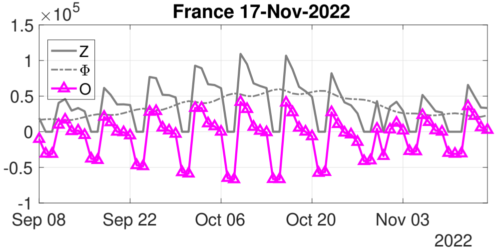
correction of corrupted Z_t & estimation of regularized R_t

(Pascal et al., 2022, *Trans. Sig. Process.*)

Extended Cori Model: additional latent variable O_t accounting for misreport

$$Z_t \sim \text{Poiss}\left(R_t \Phi_t + O_t\right), \quad R_t \Phi_t + O_t \geq 0$$

nonzero values of O_t concentrated on specific days (Sundays, day-offs, ...)



Interpretation:

$$\text{Poiss}\left(R_t \Phi_t + O_t\right) \sim \begin{cases} \text{Poiss}\left(R_t \Phi_t\right) + \text{Poiss}\left(O_t\right) & \text{if } O_t \geq 0, \\ \text{Poiss}\left(\alpha_t R_t \Phi_t\right), \ \alpha_t = 1 - \frac{-O_t}{R_t \Phi_t} \in [0, 1] & \text{if } O_t < 0. \end{cases}$$

Data: reported counts $\mathbf{Z} = (Z_1, \ldots, Z_T)$

Model: corrected Poisson $\quad \mathbb{P}(Z_t | \mathbf{Z}_{t-\tau_\Phi:t-1}, R_t, O_t) = \dfrac{(R_t \Phi_t + O_t)^{Z_t} \mathrm{e}^{-(R_t \Phi_t + O_t)}}{Z_t!}$

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Generalized Penalized Kullback-Leibler

$$(\widehat{\mathbf{R}}, \widehat{\mathbf{O}}) \in \underset{(\mathbf{R},\mathbf{O}) \in \mathbb{R}_+^T \times \mathbb{R}^T}{\operatorname{Argmin}} \sum_{t=1}^{T} \mathrm{d}_{\mathrm{KL}} \left(Z_t \mid R_t\Phi_t + O_t \right) + \lambda_{\mathrm{R}} \|\mathbf{D}_2 \mathbf{R}\|_1 + \iota_{\geq 0}(\mathbf{R}) + \lambda_{\mathrm{O}} \|\mathbf{O}\|_1$$

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properties of the objective function:

- sum of convex functions composed with linear operators $\Longrightarrow$ globally convex;
- feasible domain: $(\forall t, R_t \geq 0)$ & (if $Z_t > 0, R_t\Phi_t + O_t > 0$, else $R_t\Phi_t + O_t \geq 0$);
- $p_t \mapsto d_{\mathrm{KL}}\left(Z_t \,|\, p_t\right)$ is strictly-convex.

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Theorem (Pascal et al., 2022, *Trans. Sig. Process.*)

+ The minimization problem has at least one solution $(\widehat{\mathbf{R}}, \widehat{\mathbf{O}})$.
+ The estimated time-varying Poisson intensity $\widehat{p}_t = \widehat{R}_t\Phi_t + \widehat{O}_t$ is unique.

$$\underset{(\mathbf{R},\mathbf{O})\in\mathbb{R}_+^T\times\mathbb{R}^T}{\text{minimize}} \sum_{t=1}^{T} \mathrm{d}_{\mathrm{KL}}\left(\mathrm{Z}_t \,|\, \mathrm{R}_t\Phi_t + \mathrm{O}_t\right) + \lambda_{\mathrm{R}}\|\mathbf{D}_2\mathbf{R}\|_1 + \iota_{\geq 0}(\mathbf{R}) + \lambda_{\mathrm{O}}\|\mathbf{O}\|_1$$

- each term of the functional is convex;
- ℓ_1-norm and indicative function $\Longrightarrow$ nonsmooth; ✗ gradient descent
- gradient of $\mathrm{p}_t \mapsto \mathrm{d}_{\mathrm{KL}}\left(\mathrm{Z}_t \,|\, \mathrm{p}_t\right)$ is not Lipschitzian; ✗ forward-backward
- linear operator $\mathbf{D}_2 \Longrightarrow$ no explicit form for $\mathrm{prox}_{\|\mathbf{D}_2\cdot\|_1}$ ♣ need splitting

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$$\Longleftrightarrow \underset{(\mathbf{R},\mathbf{O})\in\mathbb{R}_+^T\times\mathbb{R}^T}{\text{minimize}} \quad f(\mathbf{R},\mathbf{O}|\mathbf{Z}) + h(\mathbf{A}(\mathbf{R},\mathbf{O})), \quad \mathbf{A} \text{ linear; } f, h \text{ proximable}$$

$$\mathbf{A}(\mathbf{R},\mathbf{O}) = (\lambda_{\mathrm{R}}\mathbf{D}_2\mathbf{R}, \mathbf{R}, \lambda_{\mathrm{O}}\mathbf{O})\,; \quad h\,(\mathbf{Q}_1,\mathbf{Q}_2,\mathbf{Q}_3) = \|\mathbf{Q}_1\|_1 + \iota_{\geq 0}(\mathbf{Q}_2) + \|\mathbf{Q}_3\|_1$$

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Primal-dual algorithm (Chambolle et al., 2011, *Int. Conf. Comput. Vis.*)

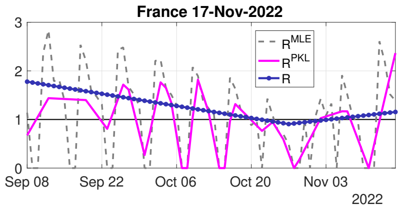
for $k = 1, 2 \ldots$ **do**

$$\mathbf{Q}^{[k+1]} = \mathrm{prox}_{\sigma h^*}(\mathbf{Q}^{[k]} + \sigma\mathbf{A}(\overline{\mathbf{R}}^{[k]}, \overline{\mathbf{O}}^{[k]})) \qquad \text{dual}$$

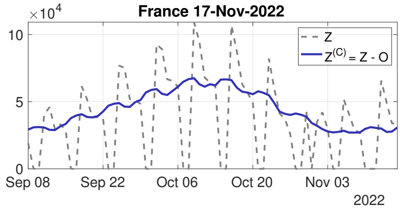
$$(\mathbf{R}^{[k+1]}, \mathbf{O}^{[k+1]}) = \mathrm{prox}_{\tau f(\cdot|\mathbf{Z})}((\mathbf{R}^{[k+1]}, \mathbf{O}^{[k+1]}) - \tau\mathbf{A}^*\mathbf{Q}^{[k+1]}) \qquad \text{primal}$$

$$(\overline{\mathbf{R}}^{[k+1]}, \overline{\mathbf{O}}^{[k+1]}) = 2(\mathbf{R}^{[k+1]}, \mathbf{O}^{[k+1]}) - (\mathbf{R}^{[k]}, \mathbf{O}^{[k]}) \qquad \text{auxiliary}$$

Reproduction number $\widehat{\mathbf{R}}$

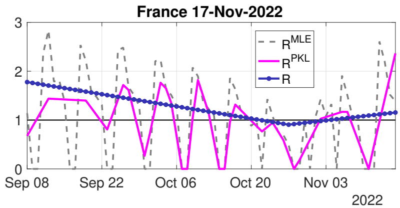


Corrected infection counts $\mathbf{Z}^{(C)}$



$\Longrightarrow$ no more pseudo-seasonality, local trends well captured, smooth behavior

Reproduction number $\widehat{\mathbf{R}}$



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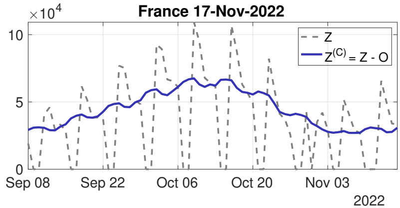

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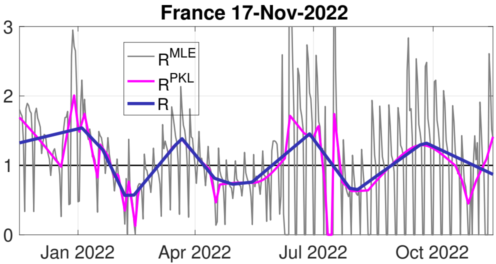

fast numerical scheme: 15 to 30 sec for 70 days to 1 year

New infection counts per county: $\mathbf{Z} = \left\{ \mathrm{Z}_t^{(d)},\, d \in [1, D],\, t \in [1,\, T] \right\}$

$\Rightarrow$ multivariate time-varying reproduction number $\mathrm{R}_t^{(d)}$

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Multivariate extended penalized Kullback-Leibler

$$\left(\widehat{\mathbf{R}}, \widehat{\mathbf{O}}\right) = \underset{(\mathbf{R},\mathbf{O}) \in \mathbb{R}_+^{D\times T} \times \mathbb{R}^{D\times T}}{\operatorname{argmin}} \sum_{d=1}^{D} \sum_{t=1}^{T} \mathsf{d}_{\mathsf{KL}}\left(\mathsf{Z}_t^{(d)} \,\middle|\, \mathsf{R}_t^{(d)} \Phi_t^{(d)} + \mathsf{O}_t^{(d)} \right)$$
$$+ \lambda_{\mathsf{R}} \|\mathbf{D}_2 \mathbf{R}\|_1 + \iota_{\geq 0}(\mathbf{R}) + \lambda_{\mathrm{space}} \|\mathbf{G}\mathbf{R}\|_1 + \lambda_{\mathsf{O}} \|\mathbf{O}\|_1$$

$\Longrightarrow$ $\|\mathbf{G}\mathbf{R}\|_1$ favors **piecewise constancy** in space

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$\Longrightarrow \|\mathbf{GR}\|_1$ favors **piecewise constancy** in space

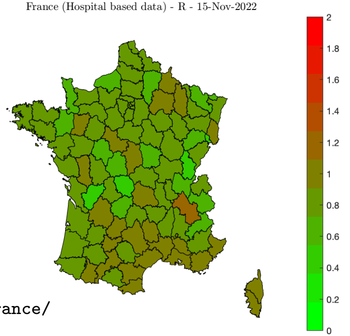
Graph Total Variation

$$\|\mathbf{GR}\|_1 = \sum_{t=1}^{T} \sum_{d_1 \sim d_2} \left| \mathsf{R}_t^{(d_1)} - \mathsf{R}_t^{(d_2)} \right|$$

sum over neighboring counties

here: $d_1 \sim d_2 \Leftrightarrow$ share terrestrial border

$$\widetilde{\mathbf{A}}(\mathbf{R}, \mathbf{O}) = (\lambda_{\mathsf{R}} \mathbf{D}_2 \mathbf{R}, \mathbf{R}, \lambda_{\text{space}} \mathbf{G}\mathbf{R}, \lambda_{\mathsf{O}} \mathbf{O})$$



`http://barthes.enssib.fr/coronavirus/cartes/RFrance/`

Bayesian framework for credibility interval estimation

Pointwise estimate of parameter $\boldsymbol{\theta} = (\mathbf{R}, \mathbf{O})$ from observations $\mathbf{Z}$

$$\underset{\boldsymbol{\theta} \in \mathbb{R}_+^T \times \mathbb{R}^T}{\text{minimize}} \quad f(\boldsymbol{\theta} | \mathbf{Z}) + h(\mathbf{A}\boldsymbol{\theta})$$

(Pascal et al., 2022, *Trans. Sig. Process.*)

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Q: what is the value of R today? **A:** solve the minimization problem and output $\widehat{\mathrm{R}}_T$.

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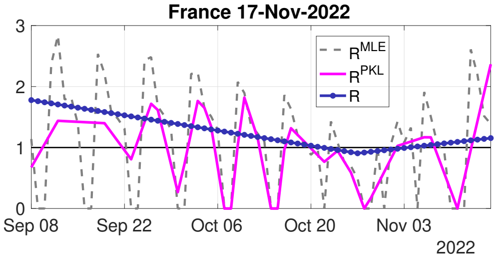

$\widehat{\mathrm{R}}_T = 1.2955$

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Bayesian reformulation: interpret $(\widehat{\mathbf{R}}, \widehat{\mathbf{O}})$ as the MAP of

$$\pi(\boldsymbol{\theta}) \propto \exp(-f(\boldsymbol{\theta}|\mathbf{Z}) - h(\mathbf{A}\boldsymbol{\theta}))$$

- $\exp(-f(\boldsymbol{\theta}|\mathbf{Z})) \sim$ likelihood of the observation
- $\exp(-h(\mathbf{A}\boldsymbol{\theta})) \sim$ prior on the parameter of interest

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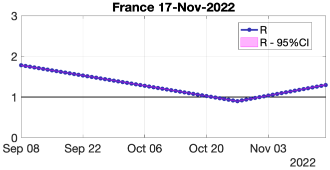
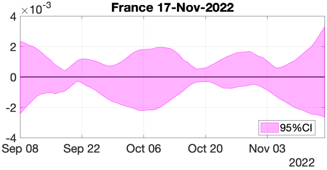
$\Longrightarrow$ instead of focusing on $\widehat{\mathrm{R}}_t$, the **pointwise** MAP, probe π to get

$\mathrm{R}_t \in [\underline{\mathrm{R}}_t, \overline{\mathrm{R}}_t]$ with 95% probability, i.e., **credibility interval** estimates





$$\widehat{\mathrm{R}}_T \in [1.2987, 1.3047]$$

Log-likelihood from Poisson model $\mathcal{D} = \{\boldsymbol{\theta} \,|\, \forall t,\ \ \mathsf{R}_t \Phi_t + \mathsf{O}_t \geq 0,\ \mathsf{R}_t \geq 0\}$

$$f(\boldsymbol{\theta}\,|\mathbf{Z}) := \begin{cases} -\sum_{t=1}^{T} (\mathsf{Z}_t \ln(\mathsf{R}_t \Phi_t + \mathsf{O}_t) - (\mathsf{R}_t \Phi_t + \mathsf{O}_t) + \mathcal{C}(\mathsf{Z}_t)) & \text{if } \boldsymbol{\theta} \in \mathcal{D}, \\ \infty & \text{otherwise,} \end{cases}$$

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Prior distribution of $\boldsymbol{\theta} = (\mathbf{R}, \mathbf{O}) = (\mathsf{R}_1, \ldots, \mathsf{R}_T, \mathsf{O}_1, \ldots, \mathsf{O}_T) \in (\mathbb{R}_+)^T \times \mathbb{R}^T$

- reproduction number: $\mathsf{R}_t - 2\mathsf{R}_{t-1} + \mathsf{R}_{t-2} \sim \mathsf{Laplace}(\lambda_{\mathsf{R}})$

Bayesian framework for credibility interval estimation

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$$\Rightarrow g(\boldsymbol{\theta}) = \lambda_{\mathsf{R}} \|\mathbf{D}_2 \mathbf{R}\|_1 + \lambda_{\mathsf{O}} \|\mathbf{O}\|_1, \quad \mathbf{D}_2 = \underbrace{\frac{1}{\sqrt{6}} \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & -2 & 1 & 0 & \ldots & 0 \\ \ldots & & & & & & \ldots \\ 0 & \ldots & & & 1 & -2 & 1 \end{bmatrix}}_{\text{Laplacian}}$$

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Posterior distribution of unknown parameters $\boldsymbol{\theta} = (\mathbf{R}, \mathbf{O})$

$$\pi(\boldsymbol{\theta}) \propto \exp\left(-f(\boldsymbol{\theta}) - g(\boldsymbol{\theta})\right) \mathbb{1}_\mathcal{D}(\boldsymbol{\theta})$$

- f, g convex
- f smooth, g nonsmooth

Markov Chain Monte Carlo sampling

Purpose: sampling the random variable $\boldsymbol{\theta} = (\mathbf{R}, \mathbf{O}) \in \mathbb{R}^{2T}$ according to the posterior[†]

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Simple and very general approach: *Hastings-Metropolis random walk*

(i) propose a random move according to

$$\boldsymbol{\theta}^{n+\frac{1}{2}} = \boldsymbol{\theta}^n + \sqrt{2\gamma}\Gamma\xi^{n+1}, \quad \xi^{n+1} \sim \mathcal{N}_{2T}(0, \mathrm{I})$$

with γ positive step size, $\Gamma \in \mathbb{R}^{2T \times 2T}$

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(ii) accept: $\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta}^{n+\frac{1}{2}}$, with probability $1 \wedge \dfrac{\pi(\boldsymbol{\theta}^{n+\frac{1}{2}})}{\pi(\boldsymbol{\theta}^n)}$, or reject: $\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta}^n$

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Langevin dynamics: $\boldsymbol{\theta}^{n+\frac{1}{2}} = \mu(\boldsymbol{\theta}^{n}) + \sqrt{2\gamma}\xi^{n+1}$, (Kent, 1978, *Adv Appl Probab*)

$$\mu(\boldsymbol{\theta}) \text{ adapted to } \pi(\boldsymbol{\theta}) = \exp(-f(\boldsymbol{\theta}) - g(\boldsymbol{\theta}))\mathbb{1}_{\mathcal{D}}(\boldsymbol{\theta})$$

Metropolis Adjusted Langevin Algorithm (MALA)

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Case 1: $g = 0$ and $-\ln \pi = f$ is smooth (Roberts & Tweedie, 1996, *Bernoulli*)

$$\mu(\boldsymbol{\theta}) = \boldsymbol{\theta} - \gamma\Gamma\Gamma^\top \nabla f(\boldsymbol{\theta}) = \boldsymbol{\theta} + \gamma\Gamma\Gamma^\top \nabla \ln \pi(\boldsymbol{\theta})$$

$\Longrightarrow$ move towards areas of higher probability

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Case 2: $-\ln \pi = f + g$ is nonsmooth

$$\mu(\boldsymbol{\theta}) = \mathrm{prox}_{\gamma g}^{\Gamma\Gamma^\top}(\boldsymbol{\theta} - \gamma\Gamma\Gamma^\top \nabla f(\boldsymbol{\theta}))$$

combining *Langevin* and *proximal*[†] approaches

[†] $\mathrm{prox}_{\gamma g}^{\Gamma\Gamma^\top}(y) = \underset{x\in\mathbb{R}^d}{\mathrm{argmin}} \left(\frac{1}{2}\|x-y\|^2_{\Gamma\Gamma^\top} + \gamma g(x)\right)$: preconditioned proximity operator of g

Posterior density of $\boldsymbol{\theta} = (\mathbf{R}, \mathbf{O})$: $\quad \pi(\boldsymbol{\theta}) \propto \exp\left(-f(\boldsymbol{\theta}) - g(\boldsymbol{\theta})\right) \mathbb{1}_{\mathcal{D}}(\boldsymbol{\theta})$

- smooth negative log-likelihood

$$\text{if } \boldsymbol{\theta} \in \mathcal{D}, \quad f(\boldsymbol{\theta}) = -\sum_{t=1}^{T} \left(\mathsf{Z}_t \ln \mathsf{p}_t(\boldsymbol{\theta}) - \mathsf{p}_t(\boldsymbol{\theta})\right), \quad \mathsf{p}_t(\boldsymbol{\theta}) = \mathsf{R}_t(\Phi \mathsf{Z})_t + \mathsf{O}_t$$

- nonsmooth convex lower-semicontinuous negative a priori log-distribution

$$g(\boldsymbol{\theta}) = \lambda_{\mathsf{R}} \|\mathbf{D}_2 \mathbf{R}\|_1 + \lambda_{\mathsf{O}} \|\mathbf{O}\|_1 = h(\mathbf{A}\boldsymbol{\theta})$$

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Proximal-Gradient dual sampler PGdual

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Case 3: $-\ln \pi = f + h(\mathbf{A}\cdot)$ (Fort et al., 2022, *preprint*)

closed-form expression of $\mathrm{prox}_{\gamma h}$ but not of $\mathrm{prox}_{\gamma h(\mathbf{A}\cdot)}$

1) extend $\mathbf{A}$ into **invertible** $\overline{\mathbf{A}}$, and h in $\bar{h}$ such that $\bar{h}(\overline{\mathbf{A}}\boldsymbol{\theta}) = h(\mathbf{A}\boldsymbol{\theta})$

2) reason on the **dual** variable $\tilde{\boldsymbol{\theta}} = \overline{\mathbf{A}}\boldsymbol{\theta}$

Langevin: drift toward higher probability regions

$$\underset{\boldsymbol{\theta}\in\mathbb{R}^{2T}}{\operatorname{argmax}} \ln \pi(\boldsymbol{\theta}) = \underset{\boldsymbol{\theta}\in\mathbb{R}^{2T}}{\operatorname{argmin}}\ f(\boldsymbol{\theta}) + \bar{h}(\overline{\mathbf{A}}\boldsymbol{\theta}) = \mathbf{A}^{-1} \underset{\tilde{\boldsymbol{\theta}}\in\mathbb{R}^{2T}}{\operatorname{argmin}}\ f(\overline{\mathbf{A}}^{-1}\tilde{\boldsymbol{\theta}}) + \bar{h}(\tilde{\boldsymbol{\theta}})$$

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Two strategies to extend $\mathbf{A} = \begin{pmatrix} \mathbf{D}_2 & 0 \\ 0 & \mathbf{I} \end{pmatrix} \in \mathbb{R}^{(2T-1)\times 2T}$ into $\overline{\mathbf{A}} = \begin{pmatrix} \overline{\mathbf{D}} & 0 \\ 0 & \mathbf{I} \end{pmatrix} \in \mathbb{R}^{2T\times 2T}$:

Langevin: drift toward higher probability regions

$$\underset{\boldsymbol{\theta}\in\mathbb{R}^{2T}}{\operatorname{argmax}} \ln \pi(\boldsymbol{\theta}) = \underset{\boldsymbol{\theta}\in\mathbb{R}^{2T}}{\operatorname{argmin}}\ f(\boldsymbol{\theta}) + \bar{h}(\overline{\mathbf{A}}\boldsymbol{\theta}) = \mathbf{A}^{-1} \underset{\tilde{\boldsymbol{\theta}}\in\mathbb{R}^{2T}}{\operatorname{argmin}}\ f(\overline{\mathbf{A}}^{-1}\tilde{\boldsymbol{\theta}}) + \bar{h}(\tilde{\boldsymbol{\theta}})$$

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Two strategies to extend $\mathbf{A} = \begin{pmatrix} \mathbf{D}_2 & 0 \\ 0 & \mathsf{I} \end{pmatrix} \in \mathbb{R}^{(2T-1)\times 2T}$ into $\overline{\mathbf{A}} = \begin{pmatrix} \overline{\mathbf{D}} & 0 \\ 0 & \mathsf{I} \end{pmatrix} \in \mathbb{R}^{2T\times 2T}$:

Invert

$$\overline{\mathbf{D}}_2 := \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -2/\sqrt{5} & 1/\sqrt{5} & 0 & \cdots & 0 \\ & \mathbf{D}_2 & & & \end{bmatrix}$$

Langevin: drift toward higher probability regions

$$\underset{\boldsymbol{\theta}\in\mathbb{R}^{2T}}{\text{argmax}}\ \ln\pi(\boldsymbol{\theta}) = \underset{\boldsymbol{\theta}\in\mathbb{R}^{2T}}{\text{argmin}}\ f(\boldsymbol{\theta}) + \bar{h}(\overline{\mathbf{A}}\boldsymbol{\theta}) = \mathbf{A}^{-1}\underset{\tilde{\boldsymbol{\theta}}\in\mathbb{R}^{2T}}{\text{argmin}}\ f(\overline{\mathbf{A}}^{-1}\tilde{\boldsymbol{\theta}}) + \bar{h}(\tilde{\boldsymbol{\theta}})$$

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Ortho

$$\overline{\mathbf{D}}_o := \begin{bmatrix} v_1^\top \\ v_2^\top \\ \mathbf{D}_2 \end{bmatrix} \qquad \begin{matrix} v_1, v_2 \in \mathbb{R}^{2T} \\ v_1 \perp v_2,\ v_1, v_2 \in (\mathbf{D}_2^\top)^\perp \end{matrix}$$

Proximal-Gradient dual sampler `PGdual`

Langevin: drift toward higher probability regions

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Proposed `PGdual` **drift terms** on $\boldsymbol{\theta} = (\mathbf{R}, \mathbf{O})$:

reproduction numbers $\mu_\mathrm{R}(\boldsymbol{\theta}) = \overline{\mathbf{D}}^{-1} \operatorname{prox}_{\gamma_\mathrm{R}\lambda_\mathrm{R}\|(\cdot)_{3:T}\|_1}\left(\overline{\mathbf{D}}\,\mathbf{R} - \gamma_\mathrm{R}\overline{\mathbf{D}}^{-\top}\nabla_\mathrm{R} f(\boldsymbol{\theta})\right)$

outliers $\mu_\mathrm{O}(\boldsymbol{\theta}) = \operatorname{prox}_{\gamma_\mathrm{O}\lambda_\mathrm{O}\|\cdot\|_1}(\mathbf{O} - \gamma_\mathrm{O}\nabla_\mathrm{O} f(\boldsymbol{\theta}))$

Data: $\overline{\mathbf{D}} = \overline{\mathbf{D}}_2$ (`Invert`) or $\overline{\mathbf{D}} = \overline{\mathbf{D}}_o$ (`Ortho`)
$\gamma_{\mathrm{R}}, \gamma_{\mathrm{O}} > 0$, $N_{\max} \in \mathbb{N}_\star$, $\boldsymbol{\theta}^0 = (\mathbf{R}^0, \mathbf{O}^0) \in \mathcal{D}$
Result: A $\mathcal{D}$-valued sequence $\{\boldsymbol{\theta}^n = (\mathbf{R}^n, \mathbf{O}^n), n \in 0, \ldots, N_{\max}\}$
for $n = 0, \ldots, N_{\max} - 1$ **do**
Sample $\xi_{\mathrm{R}}^{n+1} \sim \mathcal{N}_T(0, \mathrm{I})$ and $\xi_{\mathrm{O}}^{n+1} \sim \mathcal{N}_T(0, \mathrm{I})$;
Set $\mathbf{R}^{n+\frac{1}{2}} = \mu_{\mathrm{R}}(\boldsymbol{\theta}^n) + \sqrt{2\gamma_{\mathrm{R}}}\,\overline{\mathbf{D}}^{-1}\overline{\mathbf{D}}^{-\top}\,\xi_{\mathrm{R}}^{n+1}$;
$\mathbf{O}^{n+\frac{1}{2}} = \mu_{\mathrm{O}}(\boldsymbol{\theta}^n) + \sqrt{2\gamma_{\mathrm{O}}}\,\xi_{\mathrm{O}}^{n+1}$;
$\boldsymbol{\theta}^{n+\frac{1}{2}} = (\mathbf{R}^{n+\frac{1}{2}}, \mathbf{O}^{n+\frac{1}{2}})$;
Set $\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta}^{n+\frac{1}{2}}$ with probability

$$1 \wedge \frac{\pi(\boldsymbol{\theta}^{n+\frac{1}{2}})}{\pi(\boldsymbol{\theta}^n)} \frac{q_{\mathrm{R}}(\boldsymbol{\theta}^{n+\frac{1}{2}}, \boldsymbol{\theta}_{\mathrm{R}}^n)}{q_{\mathrm{R}}(\boldsymbol{\theta}^n, \boldsymbol{\theta}_{\mathrm{R}}^{n+\frac{1}{2}})} \frac{q_{\mathrm{O}}(\boldsymbol{\theta}^{n+\frac{1}{2}}, \boldsymbol{\theta}_{\mathrm{O}}^n)}{q_{\mathrm{O}}(\boldsymbol{\theta}^n, \boldsymbol{\theta}_{\mathrm{O}}^{n+\frac{1}{2}})},$$

$q_{\mathrm{R/O}}$: Gaussian kernel stemming from nonsymmetric proposal

and $\boldsymbol{\theta}^{n+1} = \boldsymbol{\theta}^n$ otherwise.

Algorithm 1: Proximal-Gradient dual: `PGdual Invert` and `PGdual Ortho`

Gaussian proposal: $\boldsymbol{\theta}^{n+\frac{1}{2}} = \mu(\boldsymbol{\theta}^n) + \sqrt{2\gamma}\Gamma\xi^{n+1}$

- random walks: $\mu(\boldsymbol{\theta}) = \boldsymbol{\theta}$

  `RW`: $\Gamma = \mathsf{I}$; `RW Invert`: $\Gamma = \overline{\mathbf{D}}_2^{-1}\overline{\mathbf{D}}_2^{-\top}$; `RW Ortho`: $\Gamma = \overline{\mathbf{D}}_o^{-1}\overline{\mathbf{D}}_o^{-\top}$

- Proximal-Gradient dual: $\mu_{\mathrm{R}}(\boldsymbol{\theta})$, $\mu_{\mathrm{O}}(\boldsymbol{\theta})$, $\Gamma = \overline{\mathbf{D}}^{-1}\overline{\mathbf{D}}^{-\top}$

  `PGdual Invert`: $\overline{\mathbf{D}} = \overline{\mathbf{D}}_2$; `PGdual Ortho`: $\overline{\mathbf{D}} = \overline{\mathbf{D}}_o$

Practical settings: $N_{\max} = 10^7$ iterations, 15 independent runs

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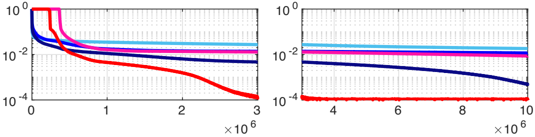
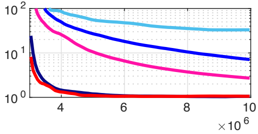
log-density

burn in | post burn in



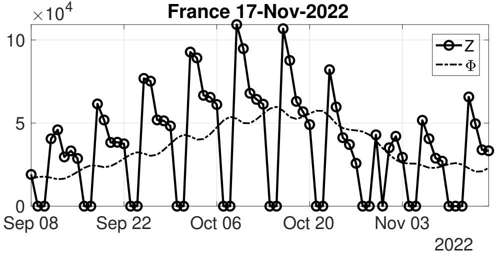
$(\ln \pi(\boldsymbol{\theta}^n) - \max \ln \pi) \,/\, \big(\ln \pi(\boldsymbol{\theta}^0) - \max \ln \pi\big)$

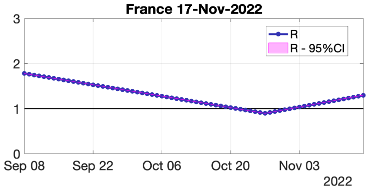
Gelman-Rubin

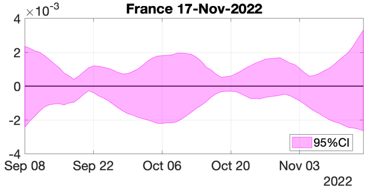


ANOVA-type criterion

Sanitary situation in France

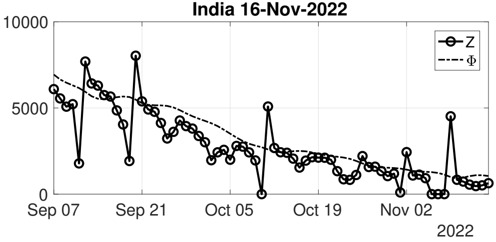


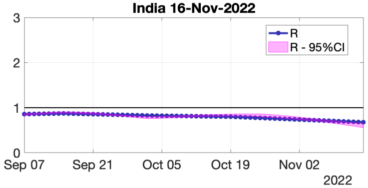


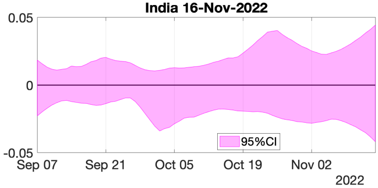


PGdual credibility interval estimation of the reproduction number

Worldwide Covid19 monitoring







Why not United Kingdom?

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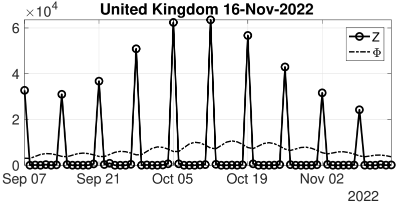
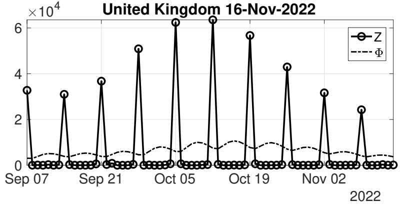


rate of erroneous counts: 6/7!

Why not United Kingdom?



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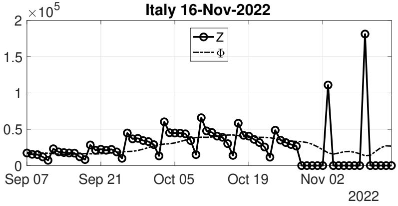
And Italy?



seems to adopt the same reporting rate ...

$\Longrightarrow$ call for new tools, robust to very scarce data

✓ Extended Cori model handling erroneous reported counts via a latent variable

$$\mathsf{Z}_t | \mathbf{Z}_{t-\tau_\Phi:t-1}, \mathsf{R}_t, \mathsf{O}_t \sim \mathsf{Poiss}(\mathsf{R}_t \Phi_t + \mathsf{O}_t)$$

Conclusion

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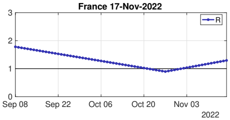
✓ Estimation of piecewise linear R_t and corrected counts via convex optimization

$$\underset{(\mathbf{R},\mathbf{O}) \in \mathbb{R}_+^T \times \mathbb{R}^T}{\text{minimize}} \sum_{t=1}^{T} \mathrm{d}_{\mathrm{KL}} \left(\mathrm{Z}_t \,|\, \mathrm{R}_t \Phi_t + \mathrm{O}_t \right) + \lambda_{\mathrm{R}} \|\mathbf{D}_2 \mathbf{R}\|_1 + \iota_{\geq 0}(\mathbf{R}) + \lambda_{\mathrm{O}} \|\mathbf{O}\|_1$$



$\widehat{\mathrm{R}}_T = 1.1959$

(Pascal et al., 2022, *Trans. Sig. Process.*;)

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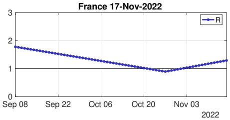
✓ Bayesian credibility interval estimates via proximal Langevin MCMC samplers



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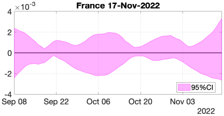

$\widehat{\mathrm{R}}_T \in [1.1978, 1.2016]$

(Pascal et al., 2022, *Trans. Sig. Process.*; Fort et al., 2022, *arXiv:2203.09142*)

Perspectives

$\longrightarrow$ Avoid mixing errors O_t with the pandemic mechanism $R_t\Phi_t$: anomaly models

$$Z_t|\mathbf{Z}_{t-\tau_\Phi:t-1}, R_t, O_t \sim \text{Poiss}((1-e_t)R_t\Phi_t + e_tO_t), \quad e_t \in \{0,1\}$$

Perspectives

⟶ Avoid mixing errors O_t with the pandemic mechanism $\mathsf{R}_t\Phi_t$: anomaly models

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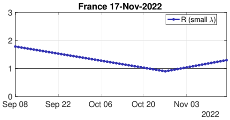
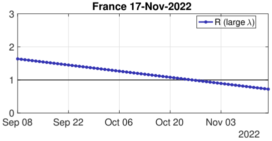
⟶ Selection of regularization parameters λ_{R}, λ_{O}

$$\underset{(\mathbf{R},\mathbf{O})\in\mathbb{R}_+^T\times\mathbb{R}^T}{\mathrm{minimize}} \ \sum_{t=1}^{T} \mathrm{d}_{\mathrm{KL}}\left(\mathsf{Z}_t \,|\, \mathsf{R}_t\Phi_t+\mathsf{O}_t\right)+\lambda_{\mathrm{R}}\|\mathbf{D}_2\mathbf{R}\|_1+\iota_{\geq 0}(\mathbf{R})+\lambda_{\mathrm{O}}\|\mathbf{O}\|_1$$





Juliana Du PhD thesis

⟶ Synthetic data