Multi-resolution in image analysis

Mathematics and Machine Learning for Image Analysis

Lab session on fractal textures

Prerequisite: Download the following toolbox and demo for lab session automated data-driven image reconstruction, including segmentation of isotropic fractal textures bpascal-fr/gsugar and isotropic_fractal_textures.m.

Automated segmentation of isotropic fractal textures

1. Generate homogeneous fractal textures corresponding to fractional Gaussian fields

$$\mathsf{G}_H(\underline{x}) = \frac{1}{2}\underbrace{(\mathsf{B}_H(\underline{x}+\underline{\mathrm{e}}_1) - \mathsf{B}_H(\underline{x}))}_{\text{horizontal increment}} + \frac{1}{2}\underbrace{(\mathsf{B}_H(\underline{x}+\underline{\mathrm{e}}_2) - \mathsf{B}_H(\underline{x}))}_{\text{vertical increment}} \text{ with } \mathsf{B}_H(\underline{x}) = \frac{\sigma}{\sqrt{\mathcal{C}_H}}\int_{\mathbb{R}^2} \frac{\mathrm{e}^{-\mathrm{i}\langle \underline{x},\underline{\xi}\rangle} - 1}{\|\underline{\xi}\|^{H+1}} \widetilde{\mathsf{W}}(\mathrm{d}\underline{\xi})$$

   - of *local variance* $\sigma^2 = 1$
   - and *local regularity*, a.k.a. *Hurst exponent*, $H \in \{0.1, 0.25, 0.5, 0.75, 0.9\}$.

   Comment on their compared visual aspects.

2. Generate a piecewise homogeneous texture composed of two regions using one of the three proposed geometries

   - "central ellipse",
   - "four ellipses",
   - "central rectangle"

   implemented in bpascal-fr/gsugar toolbox,

   - the background texture being characterized by $(H_1, \sigma_1^2) = (0.5, 1)$
   - the foreground texture by $(H_2, \sigma_2^2) = (0.75, 10)$.

3. Perform the multiscale analysis of the resulting image from scale $a_{\min} = 2$, i.e., $j_{\min} = 1$, to scale $a_{\max} = 8$, i.e., $j_{\max} = 3$, using leader wavelet coefficients

$$\mathcal{L}_{j,\underline{n}}[\mathsf{X}] = \sup_{\substack{m = \{1,2,3\} \\ \lambda_{j',\underline{n}'} \subset 3\lambda_{j,\underline{n}}}} \left| 2^{j\gamma} \zeta^{(m)}_{j',\underline{n}'}[\mathsf{X}] \right|, \text{ with } \begin{cases} \lambda_{j,\underline{n}} = [\underline{n}, \underline{n} + 2^j[\\ 3\lambda_{j,\underline{n}} = \bigcup_{\underline{p} \in \{-2^j, 0, 2^j\}^2} \lambda_{j,\underline{n}+\underline{p}}, \end{cases}$$

   and compute the linear regression estimate $\widehat{\mathrm{h}}^{\mathrm{LR}}$ of the local regularity map. Comment the quality of the estimate and its usability or not to perform segmentation.

4. Perform the estimation of local regularity map and texture segmentation using the ROF procedure

$$\underset{\mathrm{h}}{\operatorname{argmin}} \|\mathrm{h} - \widehat{\mathrm{h}}^{\mathrm{LR}}\|^2 + \theta \|\mathrm{Dh}\|_{2,1} \text{ with } \mathrm{Dh} = [\mathrm{D}_1\mathrm{h}, \mathrm{D}_2\mathrm{h}]$$

   with different regularization parameters θ ranging from 0.5 to 10 and comment on the obtained local regularity maps and segmentations.

5. Run the BFGS algorithm automatically selecting the regularization parameter by minimizing the asymptotically unbiased projected risk estimate

$$\widehat{\theta}^{\dagger} \in \underset{\theta \in \mathbb{R}_+}{\operatorname{Argmin}} \widehat{R}_{\nu,\varepsilon}(\mathcal{L}; \theta | \mathcal{S}),$$

   give the value of the optimal parameter $\theta^{\dagger}$, display the associated optimal local regularity map and segmentation and compare them to the estimates you obtained by manual tuning of the regularization parameter to assess the optimality of $\widehat{\theta}^{\dagger}$.

6. Now, run the joint and coupled segmentation strategies in the same configuration

- background texture characterized by $(H_1, \sigma_1^2) = (0.5, 1)$
- foreground texture by $(H_2, \sigma_2^2) = (0.75, 10)$

and compare the performance of these algorithms with the original local regularity-based ROF method with automated selection of θ.

7. Keep fixed the values of H_1 and H_2 and run the automated joint and coupled segmentation procedure for progressively decreasing values of σ_2^2. Comment the evolution of the segmentation performance as the local variance gap between the two regions decreases.