Texture segmentation based on multiscale parameters

Combining local variance and local regularity into an optimization scheme

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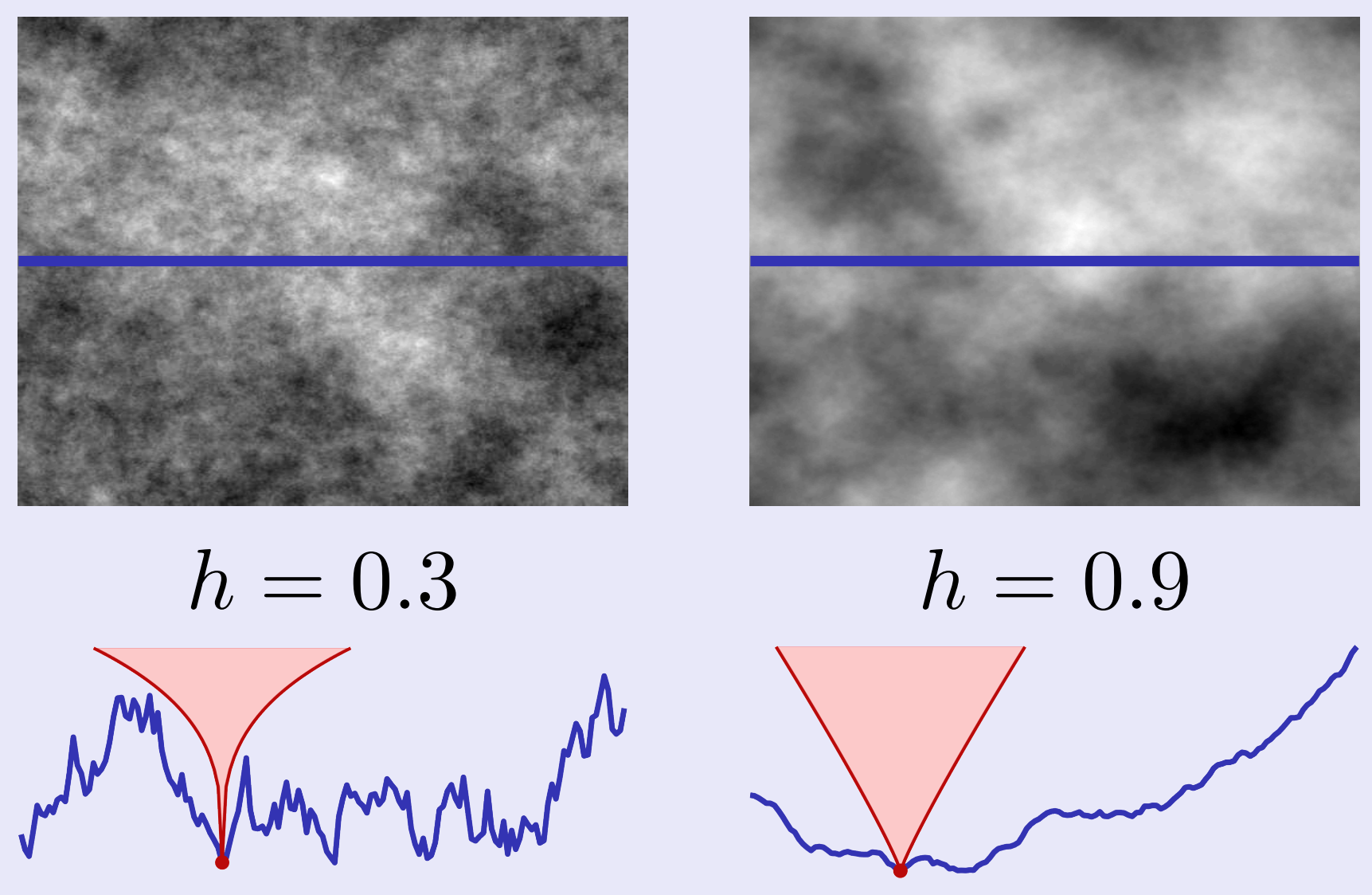
Aims and contributions

Texture segmentation constitutes a task of utmost importance in statistical image processing. Monofractal textures, characterized by piecewise constancy of their scale-free parameters, were recently shown to be versatile enough for real-world texture modeling.

Contributions
- enrolling jointly scale-free and local "variance" descriptors into a convex, but nonsmooth, minimization strategy,
- designing an efficient implementation, able to deal with huge amounts of data/high resolution images.

Numerical assessment Performance of the proposed joint approach is compared against disjoint strategies working independently on scale-free features and on local "variance" on synthetic piecewise monofractal textures.

Local regularity

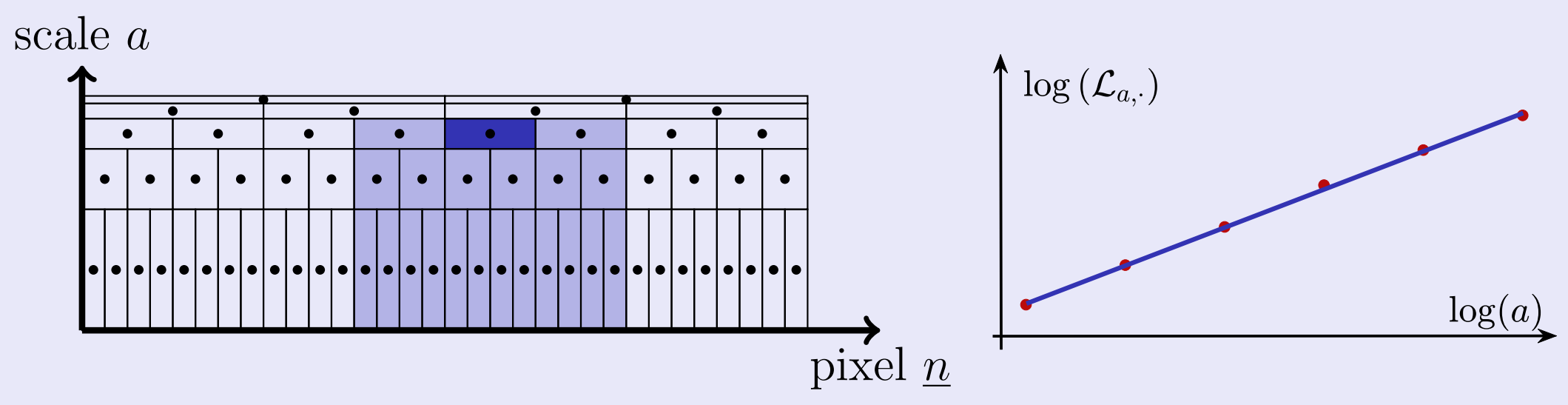


Fit local behavior with **power law** functions

$$|f(x) - f(y)| \leq C|x-y|^{h(x)}$$

Multiscale analysis

- Wavelet coefficients of the texture $w_{a,\underline{n}}$ at scale a and pixel $\underline{n}$.
- *Leaders* $\mathcal{L}_{a,\underline{n}}$: a local supremum within **a spatial neighborhood & finer scales**.
- In logarithmic coordinates *leaders* show **linear behavior w.r.t. the scale**.



$$\log\left(\mathcal{L}_{a,\underline{n}}\right) \simeq v(\underline{n}) + h(\underline{n})\log(a) \qquad \textbf{(LR)}$$

Acc. primal-dual alg.

[Chambolle, Pock, 2011]
Customized for our objective function

Primal var. $vh \underset{\text{def}}{\equiv} (v,h)$, dual var. $u\ell \underset{\text{def}}{\equiv} (u,\ell)$

```
for $k \in \mathbb{N}^*$ do
    // Update of primal variable
    $vh^{[k+1]} = \mathrm{prox}_{\delta_k \mathbf{DF}(.,\mathcal{L})}\left(vh^{[k]} - \delta_k \mathbf{D}^* \overline{u\ell}^{[k]}\right)$

    // Update of dual variable
    $u\ell^{[k+1]} = \mathrm{prox}_{\nu_k \Lambda \|\cdot\|_{2,1}^*}\left(u\ell^{[k]} + \nu_k \mathbf{D} vh^{[k]}\right)$

    // Update of descent steps
    $\vartheta_k = (1 + 2\mu\delta_k)^{-1/2}$,
    $\underbrace{\delta_{k+1} = \vartheta_k \delta_k}_{\text{smaller}}, \quad \underbrace{\nu_{k+1} = \nu_k / \vartheta_k}_{\text{larger}}$

    // Update of auxiliary variable
    $\overline{u\ell}^{[k+1]} = u\ell^{[k+1]} + \vartheta_k\left(u\ell^{[k+1]} - u\ell^{[k]}\right)$
end
```

Conclusion

Achieved:
✓ joint estimation of v and h,
✓ lead to reliable texture segmentation,
✓ with efficient implementation provided.

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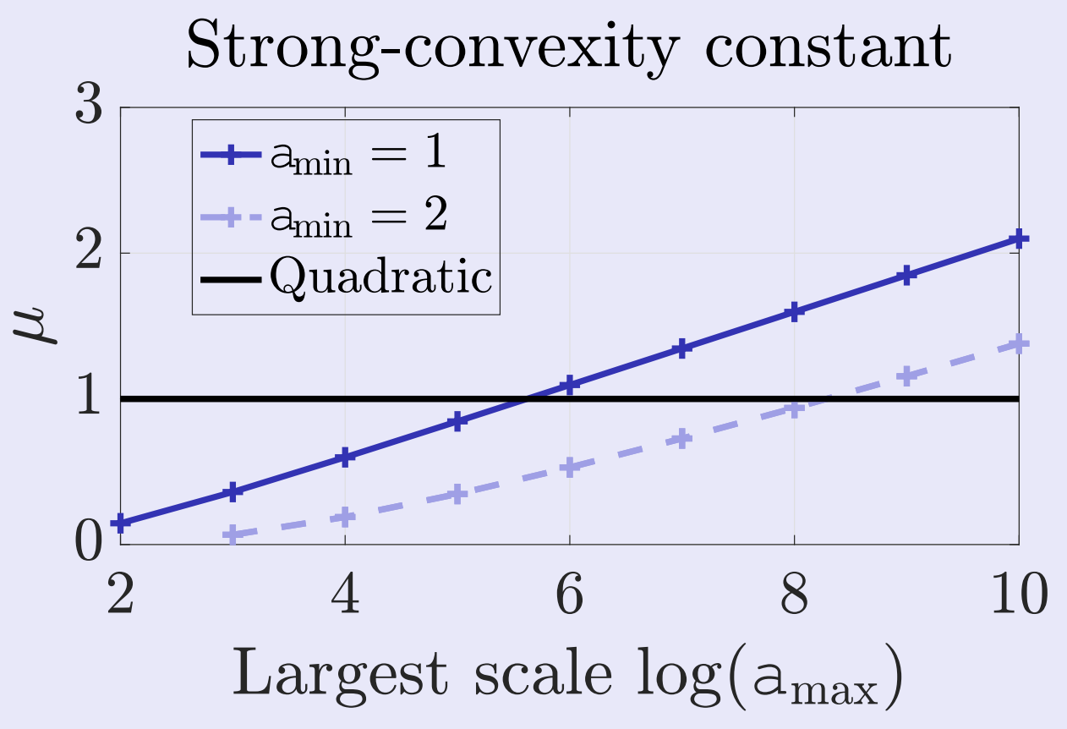
Optimization scheme

Objectives:
- match scale-free behavior **(LR)** characterizing monofractal textures,
- obtain a joint estimation of v and h directly from the *leaders* $\mathcal{L}_{a,\cdot}$,
- favor piecewise constancy of v and h without imposing same edges for v and h.

Minimization problem:

$$\left(\widehat{v}, \widehat{h}\right) \in \underset{v,h}{\mathrm{Argmin}}\ \underbrace{\mathbf{DF}(v,h\,;\,\mathcal{L}(X))}_{\text{Monofractal texture}} + \underbrace{\lambda_v \mathbf{TV}(v) + \lambda_h \mathbf{TV}(h)}_{\text{Piecewise constancy}}$$



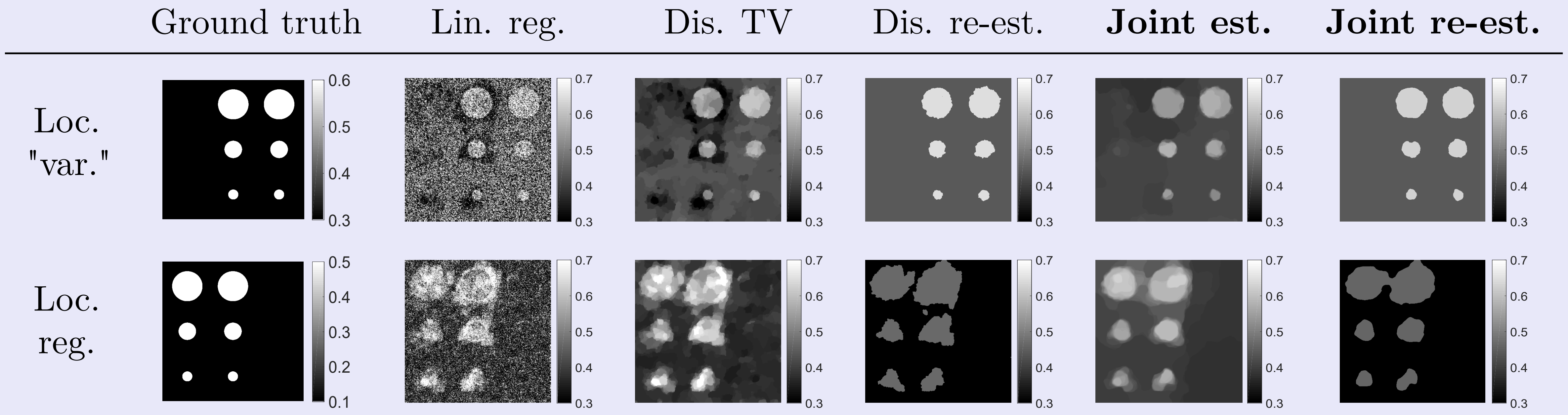
$$\mathbf{DF}(v,h\,;\,\mathcal{L}) = \frac{1}{2}\sum_{a_{\min}}^{a_{\max}} \|v + \log(a)h - \log\mathcal{L}_{a,\cdot}\|_2^2 = \frac{1}{2}\|\mathbf{A}(v,h) - \log\mathcal{L}\|_2^2$$

with $\mathbf{A} : (v,h) \mapsto \{v + \log(a)h\}_a$ **linear**. Then

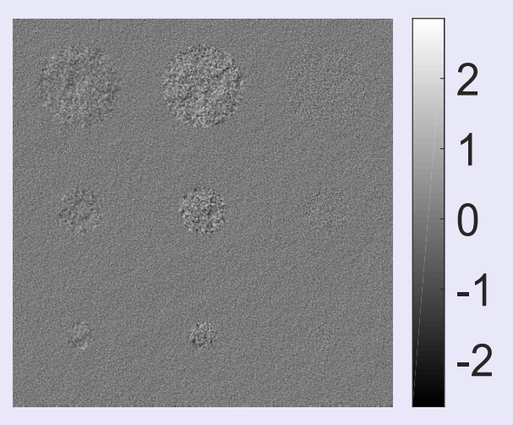
$$\nabla\mathbf{DF}(v,h\,;\,\mathcal{L}) = \underbrace{\mathbf{A}^*\mathbf{A}(v,h)}_{\text{linear}} - \underbrace{\mathbf{A}^*\log\mathcal{L}}_{\text{constant}}.$$

$\mathbf{DF}(v,h\,;\,\mathcal{L})$ is μ-strongly-convex, with $\mu > 0$ the smallest eigenvalue of $\mathbf{A}^*\mathbf{A}$

Experiments on synthetic textures

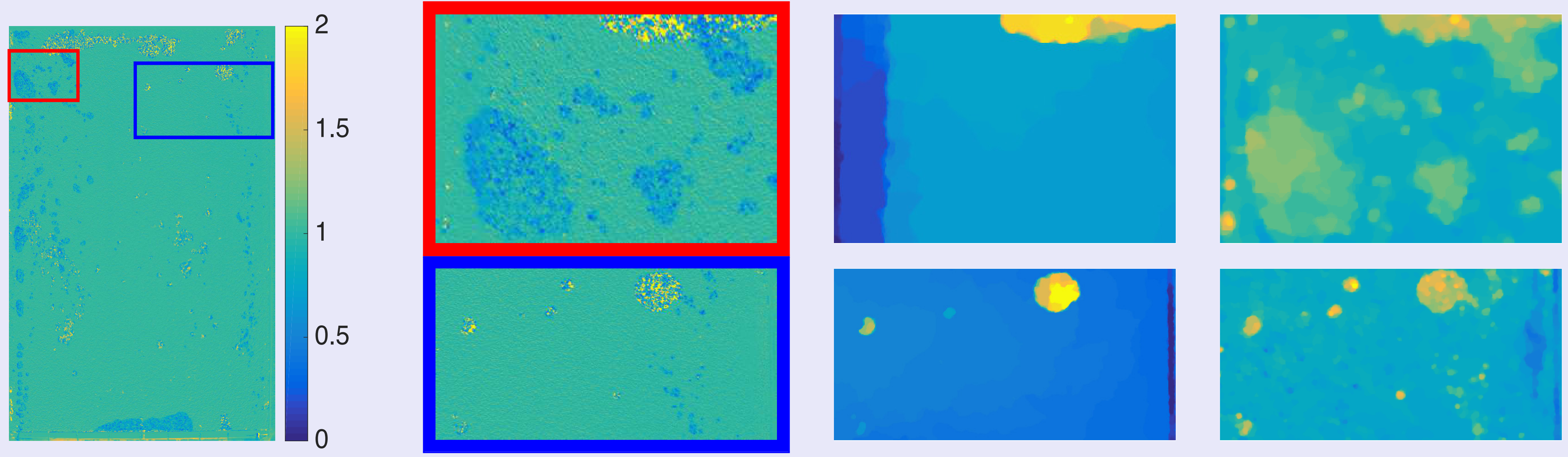


Synth. text.

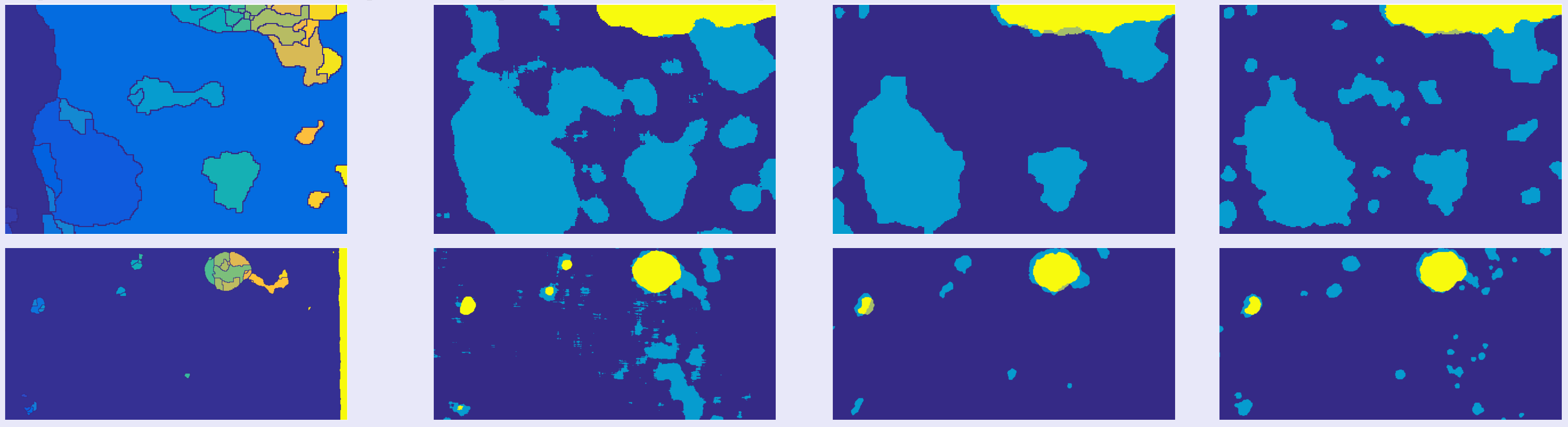


SNR(h, h_0)	-5.3411	-4.2591	0.14181	-4.1325	**0.24025**

Experiments on synthetic textures



(a) Image $x \in \mathbb{R}^{|\Omega|}$ (b) Zoom of $x \in \mathbb{R}^{|\Omega|}$ (c) Joint TV : $\widehat{v}$ (d) Joint TV : $\widehat{h}$



(e) [Arbelaez et al, 2011] (f) [Yuan et al,2015] (g) Disjoint TV (h) Joint TV