

Textured image segmentation in high dimension. Application to multiphasic flows analysis.

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Normalized image



Textured



Physical quantities: gas volume & contact surface.

area

perimeter

Texture segmentation



Texture segmentation





Texture segmentation







Purpose: obtaining a partition of the image into two regions $\Omega = \Omega_1 \bigsqcup \Omega_2$ $\Omega_1: \text{ liquid, } \Omega_2: \text{ gas.}$

Piecewise monofractal textures Local characterization





Piecewise monofractal textures

Local characterization





Texture's attributes (mathematical model)

Variance σ^2

amplitude of variations

Piecewise monofractal textures

Local characterization



Texture's attributes (mathematical model)

Variance σ^2

Local regularity h

scale-free behavior

amplitude of variations



 $h(x) \equiv h_1 = 0.9$ $h(x) \equiv h_2 = 0.3$

Fit local behavior with power law functions

 $|f(x) - f(y)| \le C|x - y|^{h(x)}$



Piecewise monofractal textures

Local characterization



Texture's attributes (mathematical model)

Variance σ^2 amplitude of variations

Local regularity *h* scale-free behavior



$$(\sigma_1^2, h_1)$$

 $h(x) \equiv h_1 = 0.9$

 $h(x) \equiv h_2 = 0.3$

Fit local behavior with power law functions

$$|f(x) - f(y)| \le C|x - y|^{h(x)}$$

Segmentation requires local measurement of σ^2 and h.

Textured image



Textured image Non-linear transform of wavelet coefficients: $\mathcal{L}_{a,.}$



Textured image

Non-linear transform of wavelet coefficients: $\mathcal{L}_{a,.}$







Textured image

Non-linear transform of wavelet coefficients: $\mathcal{L}_{a,.}$







Log-log linear behavior

$$\log \left(\boldsymbol{\mathcal{L}}_{a,\cdot} \right) \simeq \underbrace{\boldsymbol{\underline{\nu}}}_{\substack{\sim \log(\boldsymbol{\sigma}^2) \\ (\text{variance})}} + \log(a) \underbrace{\boldsymbol{\underline{h}}}_{regularity}$$

Textured image

Non-linear transform of wavelet coefficients: \mathcal{L}_a .







Log-log linear behavior $\log\left(\mathcal{L}_{a,\cdot}\right)$ $\log\left(\mathcal{L}_{a,\cdot}
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Textured image

Non-linear transform of wavelet coefficients: $\mathcal{L}_{a,\cdot}$







 $a = 2^5$

Log-log linear behavior $\log(\mathcal{L}_{a,\cdot}) \simeq \underbrace{\underline{v}}_{\substack{\sim \log(\sigma^2) \\ (\text{variance})}} + \log(a) \underbrace{\underline{h}}_{regularity}$

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Linear regression Pointwise estimates

Textured image



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Linear regression Pointwise estimates



 $\frac{\mathbb{E}\log\left(\boldsymbol{\mathcal{L}}_{a,\cdot}\right)}{\frac{\boldsymbol{\nu}}{\boldsymbol{\nu}} + \log(a)} = \frac{\boldsymbol{\nu}}{\boldsymbol{\nu}} + \log(a) \frac{\boldsymbol{\underline{h}}}{\boldsymbol{\mu}}$

Pointwise linear regression is an estimation from one sample!



$$\sum_{a} \frac{\|\log \mathcal{L}_{a,.} - \mathbf{v} - \log(a)\mathbf{h}\|^{2}}{\frac{\text{least-squares}}{\text{fidelity to log-linear model}}}$$

$$\lambda \mathcal{R}(\boldsymbol{v}, \boldsymbol{h}; \alpha)$$

 $\begin{array}{l} \textbf{total variation} \\ \rightarrow \text{ enforce piecewise constancy} \end{array}$



$$\begin{array}{ll} \underset{\mathbf{v},\mathbf{h}}{\text{minimize}} & \sum_{a} \frac{\|\log \mathcal{L}_{a,.} - \mathbf{v} - \log(a)\mathbf{h}\|^2}{|\text{east-squares}} & + & \lambda \frac{\mathcal{R}(\mathbf{v},\mathbf{h};\alpha)}{|\text{total variation}} \\ \rightarrow & \text{fidelity to log-linear model} & \rightarrow & \text{enforce piecewise constancy} \\ & & & & \\ & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ & & & & & & & \\ &$$



joint: **v**, **h** are **independently** piecewise constant

coupled: v, h are concomitantly piecewise constant



Discrete differences Hx (horizontal), Vx (vertical) at each pixel

joint: v, h are independently piecewise constant

$$\mathcal{R}_{\mathsf{J}}(\boldsymbol{v},\boldsymbol{h};\alpha) = \left(\sum_{\mathsf{pixels}} \sqrt{(\mathsf{H}\boldsymbol{v})^2 + (\mathsf{V}\boldsymbol{v})^2} + \alpha \sum_{\mathsf{pixels}} \sqrt{(\mathsf{H}\boldsymbol{h})^2 + (\mathsf{V}\boldsymbol{h})^2}\right)$$

coupled: v, h are concomitantly piecewise constant

$$\mathcal{R}_{\mathsf{C}}(\mathbf{v},\mathbf{h};\alpha) = \sum_{\mathsf{pixels}} \sqrt{(\mathsf{H}\mathbf{v})^2 + (\mathsf{V}\mathbf{v})^2 + \alpha^2(\mathsf{H}\mathbf{h})^2 + \alpha^2(\mathsf{V}\mathbf{h})^2}$$



joint: **v**, **h** are **independently** piecewise constant

$$\mathcal{R}_{\mathsf{J}}(\mathbf{v}, \mathbf{h}; \alpha) = \mathcal{R}(\mathbf{v}) + \alpha \mathcal{R}(\mathbf{h})$$

<u>coupled</u>: **v**, **h** are **concomitantly** piecewise constant

$$\mathcal{R}_{\mathsf{C}}(\mathbf{v}, \mathbf{h}; \alpha) = \mathcal{R}(\mathbf{v}, \alpha \mathbf{h})$$

Regularization parameters



Regularization parameters



Fine tuning of regularization parameters (λ, α) is necessary ...



Regularization parameters



Fine tuning of regularization parameters (λ, α) is necessary ... but **costly**!



In practice, we explore a log-spaced grid of $15 \times 15 = 225$ hyperparameters (λ, α) .

$$\underset{\boldsymbol{\nu},\boldsymbol{h}}{\text{minimize}} \sum_{\boldsymbol{a}} \frac{\|\log \mathcal{L}_{\boldsymbol{a},.} - \boldsymbol{\nu} - \log(\boldsymbol{a})\boldsymbol{h}\|^2}{\text{least-squares}} + \frac{\lambda}{\text{total variation}} \frac{\mathcal{R}(\boldsymbol{\nu},\boldsymbol{h};\alpha)}{\text{total variation}}$$

$$\underset{\mathbf{v},\mathbf{h}}{\text{minimize}} \sum_{\mathbf{a}} \frac{\|\log \mathcal{L}_{\mathbf{a},.} - \mathbf{v} - \log(\mathbf{a})\mathbf{h}\|^2}{\text{least-squares}} + \frac{\lambda}{\text{total variation}} \mathcal{R}(\mathbf{v},\mathbf{h};\alpha)$$

 \rightarrow non-smooth



primal-dual algorithm (Chambolle, Pock 11')



Accelerated primal-dual algorithm (Chambolle, Pock 11')

$$\begin{array}{ll} \underset{\mathbf{v},\mathbf{h}}{\operatorname{minimize}} & \sum_{a} \frac{\|\log \mathcal{L}_{a,.} - \mathbf{v} - \log(a)\mathbf{h}\|^2}{|\operatorname{least-squares}} & + & \lambda \underbrace{\mathcal{R}(\mathbf{v},\mathbf{h};\alpha)}_{\operatorname{total variation}} \\ \varphi \text{ is } \alpha \text{-strongly convex iff} \\ \varphi - \frac{\alpha}{2} \|\cdot\|^2 \text{ is convex.} \end{array}$$

Accelerated primal-dual algorithm (Chambolle, Pock 11')

$$\mathbf{y}^{n+1} = \operatorname{prox}_{\sigma_n \|\cdot\|_{2,1}} \left(\mathbf{y}^n + \sigma_n \nabla \bar{\mathbf{x}}^n \right)$$
$$\mathbf{x}^{n+1} = \operatorname{prox}_{\tau_n \|\mathbf{A} - \mathbf{b}\|_2^2} \left(\mathbf{x}^n - \tau_n \nabla^* \mathbf{y}^{n+1} \right)$$
$$\theta_n = \sqrt{1 + 2\mathbf{\alpha}\tau_n}, \quad \tau_{n+1} = \tau_n / \theta_n, \quad \sigma_{n+1} = \theta_n \sigma_n$$
$$\bar{\mathbf{x}}^{n+1} = \mathbf{x}^{n+1} + \theta_n^{-1} \left(\mathbf{x}^{n+1} - \mathbf{x}^n \right)$$

Segmentation of multiphasic flow images.

Comparison of joint and coupled methods to state-of-the-art and previous work.

Factorization-based segmentation [Yuan *et al.* 15'][†]

(i) local spectral histograms



(ii) matrix factorization



Fig. 2. Scatterplot of features in subspace. (a) Scatterplot of features projected onto the 3-d subspace. (b) Scatterplot after removing features with high edgeness.

[†]https://sites.google.com/site/factorizationsegmentation/

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Threshold-ROF on \hat{h}^{LR} [Pustelnik 16']

$$\min_{\boldsymbol{h}} \|\boldsymbol{h} - \widehat{\boldsymbol{h}}^{\mathrm{LR}}\|^2 + \lambda \|\boldsymbol{\nabla}\boldsymbol{h}\|_{2,1}$$









Based on regularity **h** only.

[†]https://sites.google.com/site/factorizationsegmentation/

Gas/liquid flow modeled by piecewise monofractal textures

Synthetic textures

Liquid: $h_1 = 0.4, \sigma_1^2 = 10^{-2}$



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Liquid:
$$h_1 = 0.4, \ \sigma_1^2 = 10^{-2}$$

Gas: $\begin{vmatrix} h_2 = 0.9, \ \sigma_1^2 = 10^{-2} \ (dark \ bubbles) \end{vmatrix}$



$\ensuremath{\mathsf{Gas}}\xspace/\ensuremath{\mathsf{Iiquid}}\xspace$ flow modeled by piecewise monofractal textures

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Segmentation performance



 $Multiphasic \ flow. \ \ {\it Q}_{\rm G} = 300 mL/min \ - \ {\it Q}_{\rm L} = 300 mL/min: \ low \ activity$



$Multiphasic \ flow. \ \ {\it Q}_{\rm G} = 400 mL/min \ \ - \ {\it Q}_{\rm L} = 700 mL/min: \ transition$



Multiphasic flow. $\mathit{Q}_{\rm G} = 1200 \textrm{mL}/\textrm{min}$ - $\mathit{Q}_{\rm L} = 300 \textrm{mL}/\textrm{min}$: high activity



Liquid/Gas	Clear/Dark bubbles	Smooth
(regularity change)	(variance change)	contours

	Liquid/Gas	Clear/Dark bubbles	Smooth
	(regularity change)	(variance change)	contours
Yuan	×	\checkmark	1

	Liquid/Gas (regularity change)	Clear/Dark bubbles (variance change)	Smooth contours
Yuan	×	✓	✓
T-ROF	 Image: A second s	✓	×

	Liquid/Gas (regularity change)	Clear/Dark bubbles (variance change)	Smooth contours
Yuan	×	✓	✓
T-ROF	 Image: A second s	✓	×
Joint	 Image: A set of the set of the	✓	~

	Liquid/Gas (regularity change)	Clear/Dark bubbles (variance change)	Smooth contours
Yuan	×	✓	1
T-ROF	1	✓	×
Joint	1	✓	~
Coupled	1	\checkmark	1

Comparison of the different methods

	Liquid/Gas (regularity change)	Clear/Dark bubbles (variance change)	Smooth contours
Yuan	×	\checkmark	✓
T-ROF	1	✓	×
Joint	1	✓	~
Coupled	✓	\checkmark	 Image: A second s

Coupled is the most satisfactory in term of segmentation quality ...

Comparison of the different methods

	Liquid/Gas (regularity change)	Clear/Dark bubbles (variance change)	Smooth contours
Yuan	×	\checkmark	1
T-ROF	1	✓	X
Joint	1	✓	~
Coupled	 Image: A second s	✓	1

Coupled is the most satisfactory in term of segmentation quality ...

... but it is the most time consuming (2100s) Yuan(1s), T-ROF (12s), Joint (700s)

• Video analysis (temporal series of hundreds of images)

Intership of L. Helmlinger

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✓ Best (λ, α) tuned on 1st image is sufficiently robust for the entire series.

• Video analysis (temporal series of hundreds of images)

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Intership of L. Helmlinger
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- ✓ Best (λ, α) tuned on 1st image is sufficiently robust for the entire series.
- Time evolution of physical quantities can be assessed.



Liquid/gas contact perimeter



0

0

• Video analysis (temporal series of hundreds of images)

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Intership of L. Helmlinger
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Best (λ, α) tuned on 1st image is sufficiently robust for the entire series.

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Automatic tuning of hyperparameters

Stein's Unbiased Risk Estimate $\widehat{R}(\lambda, \alpha)$ Stein Unbiased GrAdient estimator of the Risk $\nabla_{\lambda}\widehat{R}(\lambda, \alpha)$



Thank you for your attention!

Fully Convolutional Neural Networks[†]



[†] V. Andrearczyk, https://arxiv.org/abs/1703.05230

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